Week 10 Recitation

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- Go over homework from last week.
  - Make sure to show work, especially on questions: 8.4.26a (Algorithm 1), 8.4.28a (Warshall Algorithm). See handout given in recitation for an example solution to these questions
- Questions about lecture / homework so far?
- Rosen 8.5.3

  To be an equivalence relation we need to be:
  - Reflexive
  - Symmetric
  - Transitive

  We are checking to see if these functions are equivalent, using different conditions in each part.

  a) Checking: $\{f, g)|f(1) = g(1)\}$. This means (f, g) will be included in the relation if $f(1)$ and $g(1)$ evaluate to the same term.
  This relation will be reflexive since $f(1) = f(1)$.
  This relation will be symmetric because if $f(1) = g(1)$, $g(1) = f(1)$.
  This relation will be transitive because if $f(1) = g(1)$ and $g(1) = h(1)$, $f(1) = h(1)$.
  Therefore, it is an equivalence relation.

  b) $\{(f, g)|f(0) = g(0) \vee f(1) = g(1)\}$.
  This relation will *not* be transitive because if "$f(0) = g(0) \vee f(1) = g(1)$" and "$g(0) = h(0) \vee g(1) = h(1)$" it could be the case that $f(0) = g(0) \wedge f(1) \neq g(1)$ and $g(0) \neq h(0) \wedge g(1) = h(1)$, which would not make (f, h) hold.

- Rosen 8.5.41(a)

  To be a partition of the set A into non-empty set subsets $A_1, A_2, \ldots, A_k$, we need to satisfy:

  - $\cup_{i=1}^{k} A_i = A$
  - $\forall i, j | i \neq j; A_i \cap A_j = \emptyset$
  - $\forall i, A_i \neq \emptyset$

  For our partition:

  - $\cup_{i=1}^{k} A_i = \{-3, -1, 1, 3\} \cup \{-2, 0, 2\} = \{-3, -2, -1, 0, 1, -2\} = A$
  - $\{-3, -1, 1, 3\} \cap \{-2, 0, 2\} = \emptyset$
  - $\{-3, -1, 1, 3\} \neq \emptyset \wedge \{-2, 0, 2\} \neq \emptyset$

  Therefore, it is a valid partition

- Rosen 8.5.41(b)

  For our partition:

  - $\cup_{i=1}^{k} A_i = \{-3, -2, -1, 0\} \cup \{0, 1, 2, 3\} = \{-3, -2, -1, 0, 1, -2\} = A$
  - $\{-3, -2, -1, 0\} \cap \{0, 1, 2, 3\} = \{0\} \neq \emptyset$
  - $\{-3, -2, -1, 0\} \neq \emptyset \wedge \{0, 1, 2, 3\} \neq \emptyset$

  Since $\forall i, j | i \neq j; A_i \cap A_j \neq \emptyset$, it is *not* a valid partition

- Rosen 8.6.1(a)

  A relation R on a set S is a partial order if it is:

  - Reflexive
  - Antisymmetric
  - Transitive

  Note: That if R satisfies these conditions, the set S with the partial order R is called a *partially ordered set (poset)* and is denoted (S, R).

  Our relation is: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

  Our relation is reflexive, antisymmetric, and transitive. Therefore, our relation is a partial order.

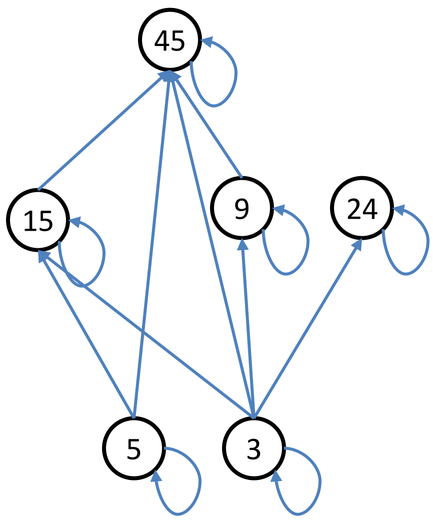
- Rosen 8.6.1 b)

  Our relation is: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

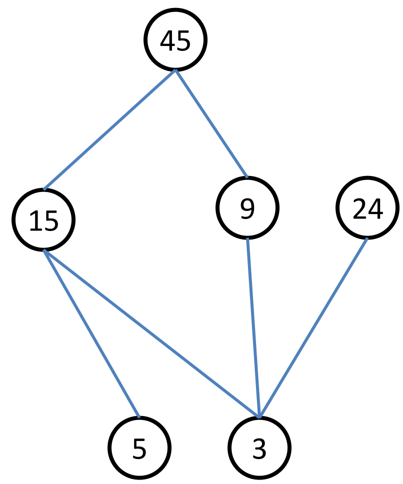
  Our relation is reflexive, and transitive, but is *not* antisymmetric. Therefore, our relation is *not* a partial order.

- Rosen 8.6:33

  First draw the relation as a directed graph, then convert it to a hasse diagram:



Directed Graph



Hasse Diagram

  Maximal elements: $\{45, 24\}$

  Minimal elements: $\{5, 3\}$

  No greatest element

  No least element

  Upper bounds of $\{3, 5\}$: $\{15, 45\}$

  Least upper bounds of $\{3, 5\}$: 15

  Lower bounds of $\{15, 45\}$: $\{15, 5, 3\}$

  Greatest lower bound of $\{15, 45\}$: 15

- Rosen 8.1.24(b)

  Complementary relation $\bar{R}$ is the set of ordered pairs $\{(a, b)|(a, b) \notin R\}$.

  $\bar{R} = \{(a, b)|a$ does not divide $b\}$

- Consider:

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\bar{H} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

- Quiz (Last 15 minutes)

Extra material:

- Rosen 8.5.1(a)

  Our relation is: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

  Our relation is reflexive, symmetric, and transitive. Therefore, it is an equivalence relation.

- Rosen 8.5.1(b)

  Our relation is: $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

  It is *not* reflexive and is *not* transitive. Therefore, it is *not* an equivalence relation.