Problem: A proof by contradiction, using cases
Prove that if $n$ is a positive integer, then $n^2 + 3n + 2$ is even.

Answer: The proof is done using contradiction and cases.
The statement to prove is the following:

If $n$ is a positive integer, then $n^2 + 3n + 2$ is even.

It is of the form $p \rightarrow q$, whose negation is $p \land \neg q$. Thus, we assume that:

- $n$ is positive integer (which $p$), and
- $n^2 + 3n + 2$ is odd (which $\neg q$).

Then, we show that the above yields a contradiction, which will prove the original statement.

$n^2 + 3n + 2$ is odd \Rightarrow
n^2 + 3n + 2$ can be written as $2m + 1$ for some integer $m$ ($m \in \mathbb{Z}$) \Rightarrow
n^2 + 3n + 2 = 2m + 1 \Rightarrow
n^2 + 3n = 2m - 1$ (moving 2 to the right-hand side) \Rightarrow
n^2 + 3n - 2m = -1$ (moving 2m to the left-hand side) \Rightarrow
n(n + 3) - 2m = -1$ (factoring out $n$)

**Case 1: $n$ is odd.** $n$ is odd \Rightarrow $(n + 3)$ is even \Rightarrow $n(n + 3)$ is even.

**Case 2: $n$ is even.** $n$ is even \Rightarrow $n(n + 3)$ is even

Thus, in all possible cases, $n(n + 3)$ is even \Rightarrow $n(n + 3) = 2k$ for some $k \in \mathbb{Z}$.

From above, we have $n^2 + 3n + 2$ is odd \Rightarrow
\[ n(n + 3) - 2m = -1 \Rightarrow 2k - 2m = -1 \Rightarrow 2(k - m) = -1 \text{ where } (k - m) \in \mathbb{Z}, \text{ which is a contradiction be it says that an even integer is equal -1.} \]

We have shown that the negation of the statement to prove yields a contradiction. In conclusion, the statement holds. \square

**Problem: Proof by cases.** Prove that \( n^4 - n^2 \) is divisible by 3 for all \( n \in \mathbb{N} \).

**Answer:** To show that \( n^4 - n^2 \) is divisible by 3, we consider three cases:

1. \( n = 3k \),
2. \( n = 3k + 1 \), and
3. \( n = 3k + 2 \),

where \( k \in \mathbb{N} \). The three cases cover all possible numbers in \( \mathbb{N} \). So, we will show the proof by cases.

**Case 1:** \( n = 3k \).

\[
\begin{align*}
n^4 - n^2 &= n^2(n^2 - 1) \\
&= (3k)^2((3k)^2 - 1) \\
&= 3(3k^2((3k)^2 - 1)).
\end{align*}
\]

Since this number is a multiple of 3, it is divisible by 3. Thus, we have shown that for \( n = 3k \), \( n^4 - n^2 \) is divisible by 3.

**Case 2:** \( n = 3k + 1 \).

\[
\begin{align*}
n^4 - n^2 &= n^2(n^2 - 1) \\
&= (3k + 1)^2((3k + 1)^2 - 1) \\
&= (3k + 1)(3k + 1)((3k + 1)(3k + 1) - 1) \\
&= (9k^2 + 6k + 1)(9k^2 + 6k + 1 - 1) \\
&= (9k^2 + 6k + 1)(9k^2 + 6k) \\
&= (9k^2 + 6k)(9k^2 + 6k + 1) \\
&= 3(3k^2 + 2k)(9k^2 + 6k + 1).
\end{align*}
\]

Since this number is a multiple of 3, it is divisible by 3. Thus, we have shown that for \( n = 3k + 1 \), \( n^4 - n^2 \) is divisible by 3.
Case 3: \( n = 3k + 2 \).

\[
\begin{align*}
    n^4 - n^2 &= n^2(n^2 - 1) \\
    &= (3k + 2)^2((3k + 2)^2 - 1) \\
    &= (3k + 2)(3k + 2)((3k + 2)(3k + 2) - 1) \\
    &= (9k^2 + 12k + 4)(9k^2 + 12k + 4 - 1) \\
    &= (9k^2 + 12k + 4)(9k^2 + 12k + 3) \\
    &= (9k^2 + 12k + 3)(9k^2 + 12k + 4) \\
    &= 3(3k^2 + 4k + 1)(9k^2 + 12k + 4).
\end{align*}
\]

Since this last number is a multiple of 3, it is divisible by 3. Thus, we have shown that for \( n = 3k + 2 \), \( n^4 - n^2 \) is divisible by 3.

Therefore, we have shown that for all cases, \( n^4 - n^2 \) is divisible by 3.

Problem: Problem 2.1.9 from textbook

Determine whether each of these statement is T or F.

a) \( x \in \{x\} \)
b) \( \{x\} \subseteq \{x\} \)
c) \( \{x\} \in \{x\} \)

Answer: Answer is in textbook.

Problem: Problem 2.1.31 from textbook

Explain why \( A \times B \times C \) and \( (A \times B) \times C \) are not the same.

Answer: Answer is in textbook.

Problem: Problem 2.2.26 from textbook

Draw the Venn diagrams for each of these combinations of the sets \( A, B, \) and \( C \).

1. \( A \cap (B \cup C) \)
2. \( \bar{A} \cap \bar{B} \cap \bar{C} \)
3. \( (A - B) \cup (A - C) \cup (B - C) \)
Answer: Answer is in the handwritten recitation notes of Week7 of the GTA.

Problem: Problem 2.2.32 from textbook
Find the symmetric difference of \{1, 3, 5\} and \{1, 2, 3\}.

Answer: Answer is in the handwritten recitation notes of Week7 of the GTA.

Problem: Problem 2.2.40 from textbook
Determine whether the symmetric different is associative; that is, if \(A, B, \text{ and } C\) are sets, does it follow that \(A \oplus (B \oplus C) = (A \oplus B) \oplus C\)?

Answer: Answer is in the handwritten recitation notes of Week7 of the GTA.