1.3. 60 (b) (c)

✓ Let $P(x)$, $Q(x)$ and $R(x)$ be the statements "x is a clear explanation," "x is satisfactory," and "x is an excuse," respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and $P(x)$, $Q(x)$ and $R(x)$.

a) All clear explanations are satisfactory.

$$\forall x \left(P(x) \to Q(x)\right)$$

b) Some excuses are unsatisfactory

$$\exists x \left(R(x) \land \neg Q(x)\right)$$

c) Some excuses are not clear explanations.

$$\exists x \left(R(x) \land \neg P(x)\right)$$

d) Does (c) follow from (a) and (b)? (YES)

prove:

conjunction

1.	~~R(a)~~ $R(a) \land \neg Q(a)$	Existential Instantiation on (b)
2.	$P(a) \to Q(a)$	U.I on (a)
3.	$\neg Q(a) \to \neg P(a)$	Contraposition on (2)
4.	$\neg Q(a)$	Simplification on (1)
5	$\neg P(a)$	Modus ponens. on (4)
6.	$R(a)$	Simplification on (1)
7.	$R(a) \land \neg P(a)$	Conjunction from (4), (6)
8.	$\exists x\ R(x) \land \neg P(x)$	Existential generalization from (7).

P67. Ex 6:

Show that the hypothesis "(It is not sunny this afternoon) and (it is colder than yesterday)" "(We will go swimming) only if it is sunny." "If we do not go swiming, then (we will take a canoe trip)". and if we take a canoe trip, then (we will be home by sun set)." lead to conclusion "we will be home by sunset".

Solution: Let p: 'It is sunny this afternoon',

q: "It is colder than yesterday"

r: "We will go swimming"

s: "We will take a canoe trip"

t: "We will be home by sunset"

Then the hypotheses become

$$\neg p \wedge q, \quad r \to p, \quad \neg r \to s, \text{ and } s \to t$$

The conclusion is t.

We construct an argument to show that our hypotheses lead to the desired conclusion as follows.

step	reason
1. $\neg p \wedge q$	Hypothesis
2. $\neg p$	Simplification using (1)
3. $r \to p$.	Hypothesis
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \to s$	Hypothesis
6. s	Modus. ponens using (4) and (5)
7. $s \to t$	Hypothesis
8. t.	Modus ponens using (6) & (7)

Note that we could have used a truth table to show that whenever each of the 4 hypotheses is true, the conclusion is also true. However, ∵ we are working with 5 proposition variables: $p.\ q.\ r.\ s.\ t$.

ii) $x+5 = (2k+1)+5 = 2k+6 = 2(k+3)$

$\therefore$ $x+5$ is even

iii) $x^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$

$\therefore$ x^2 is odd.

$\therefore$ we prove that the 3 statement are equivalent.

#.

Exe/ for Dr. Soh's hand out II:

3: Prove that the sum of ~~these~~ three consecutive integers is divisible by 3:

proof: Let the three consecutive integers be,

k, $k+1$, $k+2$.

the sum of these 3 integers is.

Sum $= k + (k+1) + (k+2) = 3k+3. = 3(k+1)$

Since $3(k+1)$ is always divisible by 3.

we have proved that the sum of three consecutive integers is divisible of 3.

Ex13. P71 ✓

Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book".

Solution: Let $C(x)$: "x in this class"

$B(x)$: "x has read the book"

x: people in the whole university

$P(x)$: "x passed the first exam"

The premises are $\exists x\,(C(x) \wedge \neg B(x))$ and $\forall x\,(C(x) \to P(x))$. The conclusion is $\exists x\,(P(x) \wedge \neg B(x))$. These steps can be used to establish the conclusion from the premises.

step	~~Sol~~ Reason.
1. $\exists x\,(C(x) \wedge \neg B(x))$	Premise
2. $C(a) \wedge \neg B(a)$	Existential instantiation from (1)
3. $C(a)$	Simplification from (2)
4. $\forall x\,(C(x) \to P(x))$	Premise.
5. $C(a) \to P(a)$	Universal instantiation from (4)
6. $P(a)$	Modus ponens from (3) & (5)
7 $\neg B(a)$	Simplification from (2)
8. $P(a) \wedge \neg B(a)$.	Conjunction from (6) & (7)
9. $\exists x\,(P(x) \wedge \neg B(x))$	Existential generalization from (8)

, ~~form~~ from Dr. soh

- show that the premise "A car in this garage has an engine problem" and "Every car in this garage has been sold" imply the conclusion "A car which has been sold ~~was~~ has an engin problem." Let $G(x)$ be "x is in this garage," $E(x)$ be "x has an engine problem" and $S(x)$ be "x has been sold." The premises are $\exists x\,(G(x) \wedge E(x))$ and $\forall x\,(G(x) \rightarrow S(x))$, The conclusion is $\exists x\,(S(x) \wedge E(x))$. ~~Fill in the following~~

proof:

1.	$\exists x\,(G(x) \wedge E(x))$	premise or Hypothesis
2.	$G(a) \wedge E(a)$	Existential instantiation from
3.	$G(a)$	Simplification from (2)
4.	$\forall x\,(G(x) \rightarrow S(x))$	Premise
5.	$G(a) \rightarrow S(a)$	Universal instantiation from (4)
6.	$S(a)$	Modus ponens. (3) & (5)
7	$E(a)$	Simplification from (2)
8	$S(a) \wedge E(a)$	Conjunction (6) & (7)
9.	$\exists x\,(S(x) \wedge E(x))$	Existential generalization from (8

Q.E.D.