Let $P(x)$, $Q(x)$, and $R(x)$ be the statements: "x is a clear explanation," "x is satisfactory," and "x is an excuse," respectively. Suppose that the domain for $x$ consists of all English text. Express each of these statements using quantifiers, logical connectives, and $P(x)$, $Q(x)$, and $R(x)$.

a) All clear explanations are satisfactory.
   \[ \forall x \ (P(x) \rightarrow Q(x)) \]

b) Some excuses are unsatisfactory
   \[ \exists x \ (R(x) \land \neg Q(x)) \]

c) Some excuses are not clear explanations.
   \[ \exists x \ (R(x) \land \neg P(x)) \]

d) Does (c) follow from (a) and (b)? \(\text{YES}\)

Proof:
1. $R(a) \land \neg Q(a)$  \(\text{Existential Instantiation on (b)}\)
2. $P(a) \rightarrow Q(a)$  \(\text{U.I. on (a)}\)
3. $\neg Q(a) \rightarrow \neg P(a)$  \(\text{Contraposition on (2)}\)
4. $\neg Q(a)$  \(\text{Simplification on (3)}\)
5. $\neg P(a)$  \(\text{Modus ponens on (4)}\)
6. $R(a)$  \(\text{Simplification on (1)}\)
7. $R(a) \land \neg P(a)$  \(\text{Conjunction from (4), (6)}\)
8. $\exists x \ (R(x) \land \neg P(x))$  \(\text{Existential generalization from (7)}\)
Show that the hypothesis "(It is not sunny this afternoon) and (it is colder than yesterday)" "We will go swimming) only if it is sunny."
"If we do not go swimming, then (we will take a canoe trip)." and if we take a canoe trip, then (we will be home by sunset)." lead to conclusion "we will be home by sunset."

Solution: Let $p$: "It is sunny this afternoon."
$q$: "It is colder than yesterday."
$r$: "We will go swimming."
$s$: "We will take a canoe trip.
$t$: "We will be home by sunset."

Then the hypotheses become

$p \land q, r \rightarrow p, r \rightarrow s, s \rightarrow t.$

The conclusion is $t.$

We construct an argument to show that our hypotheses lead to the desired conclusion as follows.
1. $p \land q$
2. $p$
3. $r \rightarrow p$
4. $r$
5. $r \rightarrow s$
6. $s$
7. $s \rightarrow t$
8. $t$. 

Note that we could have used a truth table to show that whenever each of the 4 hypotheses is true, the conclusion is also true. However, we are working with 5 proposition variables: $p, q, r, s, t$. 

Reason:
Hypothesis
Simplification using (1)
Hypothesis
Modus tollens using (2) and (3)
Hypothesis
Modus ponens using (4) and (5)
Modus ponens using (6) and (7)
ii) \[ X + 5 = (2k+1) + 5 = 2k + 6 = 3(k+2) \]

\[ \therefore X+5 \text{ is even} \]

iii) \[ X^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k + 1) \]

\[ \therefore X^2 \text{ is odd} \]

\[ \therefore \text{we prove that the 3 statements are equivalent}. \]

Example for Dr. Joh's handout:

3. Prove that the sum of three consecutive integers is divisible by 3.

Proof: Let the three consecutive integers be:

\[ k, \ k+1, \ k+2 \]

The sum of these 3 integers is:

\[ \text{Sum} = k + (k+1) + (k+2) = 3k + 3 = 3(k+1) \]

Since \( 3(k+1) \) is always divisible by 3, we have proved that the sum of three consecutive integers is divisible by 3.
Show that the premises "A student in this class has not read the book." and "Everyone in this class passed the first exam." imply the conclusion "someone who passed the first exam has not read the book."

Solution: Let $C(x)$: "x in this class"  
$B(x)$: "x has read the book"  
$P(x)$: "x passed the first exam"

The premises are $\exists x (C(x) \land \neg B(x))$ and $\forall x (C(x) \rightarrow P(x))$. The conclusion is $\exists x (P(x) \land \neg B(x))$. These steps can be used to establish the conclusion from the premises.

Step 1. $\exists x (C(x) \land \neg B(x))$  
   Reason: Premise.

Step 2. $C(a) \land \neg B(a)$  
   Reason: Existential instantiation from (1).

Step 3. $C(a)$  
   Reason: Simplification from (2).

Step 4. $\forall x (C(x) \rightarrow P(x))$  
   Reason: Premise.

Step 5. $C(a) \rightarrow P(a)$  
   Reason: Universal instantiation from (4).

Step 6. $P(a)$  
   Reason: Modus ponens from (3) and (5).

Step 7. $\neg B(a)$  
   Reason: Simplification from (2).

Step 8. $P(a) \land \neg B(a)$  
   Reason: Conjunction from (6) and (7).

Step 9. $\exists x (P(x) \land \neg B(x))$  
   Reason: Existential generalization from (8).
Proof:
1. \( \exists x (E(x) \land S(x)) \)  
2. \( G(a) \land E(a) \)  
3. \( G(a) \rightarrow S(a) \)  
4. \( \forall x (G(x) \rightarrow S(x)) \)  
5. \( G(a) \rightarrow S(a) \)  
6. \( S(a) \)  
7. \( E(a) \)  
8. \( S(c) \land E(c) \)  

Premise

Conjunction (1 & 2)

Simplification from (a)

Modus ponens (2 & 3)

Universal instantiation from (4)

Existential instantiation from (2)

Existential instantiation from (5)

Simplification from (3)

Simplification from (4)

Existential generalization from (1)

\( \exists x (G(x) \land E(x)) \) and \( \forall x (G(x) \rightarrow S(x)) \).

The conclusion is \( \exists x (S(x) \land E(x)) \).