Overview

- Defining Propositional Logic
  – Propositions
  – Connectives
  – Truth tables
- Precedence of Logical Operators
- Usefulness of Logic
  – Bitwise operations
  – Logic in Theoretical Computer Science (SAT)
  – Logic in Programming
- Logical Equivalences
  – Terminology
  – Truth tables
  – Equivalence rules

Propositions

- What is Proposition:

A declarative sentence that is either true or false, but not both. Examples:

Exercise1.1:1

- Which of these sentences are propositions? What are the truth values of those that are proposition?
  - a) Boston is the capital of Massachusetts.

    yes , T
  - b) Miami is the capital of Florida.

    yes , F

Exercise1.1:1

- Which of these sentences are propositions? What are the truth values of those that are proposition?
  - c) 2+3 =5 (yes. T)
  - d) 5+7 = 10 (yes. F)
  - e) x+2 = 11 (No.)
  - f) Answer this question. (No.)

Exercise1.1:13

- Determine whether each of these conditional statements is true or false.

  – If 1+1=2, then 2+2=5. (F)
  – If 1+1=3, then 2+2=4. (T)
  – If 1+1=3, then 2+2=5. (T)
  – If monkeys can fly, then 1+1=3 (T)

Converse, Inverse, Contrapositive

- Consider the proposition $p \rightarrow q$
  - Its converse is the proposition $q \rightarrow p$
  - Its inverse is the proposition $\neg p \rightarrow \neg q$
  - Its contrapositive is the proposition $\neg q \rightarrow \neg p$

Exercise1.1:27

- Construct a truth table for each of these compound propositions.
  - a) $p \wedge \neg q$

p	q	$p \wedge \neg q$

Exercise1.1:27

- Construct a truth table for each of these compound propositions.
  - a) $p \wedge \neg q$

p	q	$p \wedge \neg q$
0	1	0
1	0	0

Exercise1.1:27

- Construct a truth table for each of these compound propositions.
  - c) $(p \vee \neg q) \rightarrow q$

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
0	0			
0	1			
1	0			
1	1			

Exercise1.1:27

- Construct a truth table for each of these compound propositions.
  - c) $(p \vee \neg q) \to q$

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \to q$
1	1	0	1	1
1	0	1	1	0
0	1	0	0	1
0	0	1	1	0

Logical Equivalences: Ex1.2:3

- Use truth tables to verify the commutative laws.
  - a) $\boldsymbol{p \lor q \equiv q \lor p}$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{p \lor q}$	$\boldsymbol{q \lor p}$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

The two columns in the truth table are identical, thus we conclude that

$$\boldsymbol{p \lor q \equiv q \lor p}$$