$\boxed{\text{(set)}}$

P120. 9. Determine whether each of these statement is T or F.

a) $x \in \{x\}$

T

b) $\{x\} \subseteq \{x\}$

T

c) $\{x\} \in \{x\}$

F

d) $\{x\} \in \{\{x\}\}$

T

e) $\phi \subseteq \{x\}$

T

f) $\phi \in \{x\}$

F

→ ~~profer~~ proper set

$A \subset B$

if $A \neq B$.

→ prove $A \subset B$.

we needs to profe

$\begin{cases} A \text{ is a sub set of } B \\ \& \ \exists x\, (x \in B \wedge x \notin A) \end{cases}$

~~P1~~ 31. Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same?

proof: The elements of $A \times B \times C$ consists of 3-tuples (a, b, c), where $a \in A$, $b \in B$, $c \in C$.

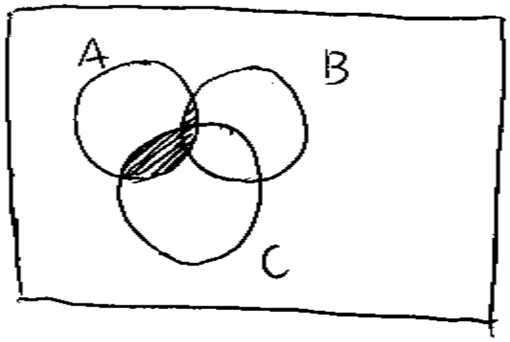
the elements of $(A \times B) \times C$ look like $((a,b), c)$, which is an ordered pair.

So, $A \times B \times C \neq (A \times B) \times C$

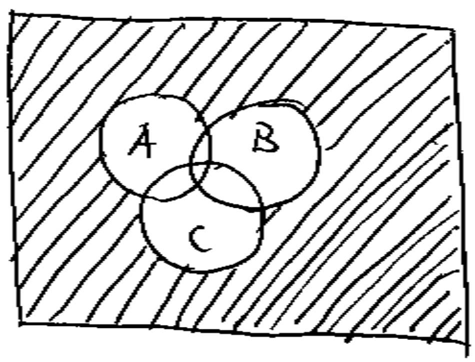
P131: 2.2.26 Draw the Venn diagrams for each of these combinations of the sets A. B. and C.

a) $A \cap (B \cup C)$

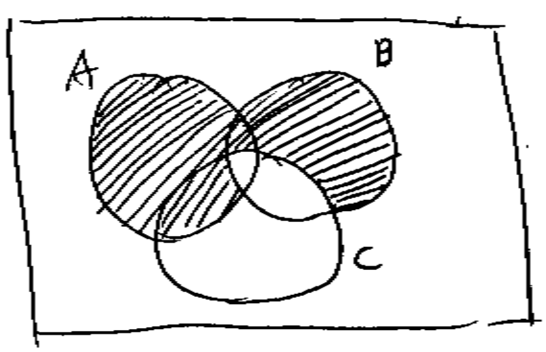
sol:



b). $\bar{A} \cap \bar{B} \cap \bar{C}$



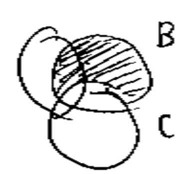
c) $(A-B) \cup (A-C) \cup (B-C)$



$A-B$

$A-C$

$B-C$



P131. 2.2.32.

Find the symmetric difference of $\overset{A}{\{1.3.5\}}$ and $\overset{B}{\{1.2.3\}}$

Sol: difference of A and B: is

$A - B$

$= \{1.3.5\} - \{1.2.3\}$

$= \{5\}$

difference of B and A is

$B - A$

$= \{1.2.3\} - \{1.3.5\}$

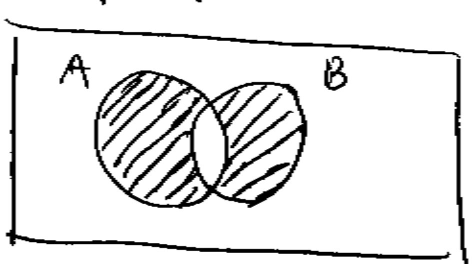
$= \{2\}$

$\therefore$ symmetric difference is $\{2, 5\}$

★ Symmetric difference of A and B. denote by $A \oplus B$.

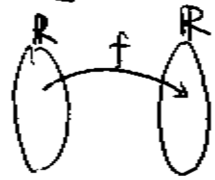
→ $A \oplus B := (A-B) \cup (B-A)$

→ Venn diagram:



(functions)

P146. 1. why is f not a function from R to R if



a) $f(x) = \frac{1}{x}$

$\because$ $f(0)$ is not defined

b) $f(x) = \sqrt{x}$

$\because$ $f(x)$ is not defined for $x<0$

c) $f(x) = \pm\sqrt{(x^2+1)}$

$\because$ $f(x)$ is not well defined because there are two distinct values assigned to each x.

A function "f" from A to B is an assignment of exactly one element of B to each element of A. We write $f(a)=b$ if b is the unique element of B assigned by the function f to the element a of A.

Problem 2: For each of functions $f: \mathbb{N} \to \mathbb{N}$, Determine whether they are one-to-one? onto?

a) $f(x) = 2x$. — one-to-one. NOT. onto.

solu: → To show f is one-to-one, we suppose $f(x_1) = f(x_2)$

by definition of $f(x)$, this means $2x_1 = 2x_2$.

Dividing both sides by 2, we get: $x_1 = x_2$

Thus. f is one-to-one.

→ To show f is not onto, we note that $1 \in \mathbb{N}$.

but we can not find an $x \in \mathbb{N}$. s.t. $f(x) = 2x = 1$.

b). $f(x) = \lfloor x/2 \rfloor$ — onto but not one-to-one.

solution: → To show f is not one-to-one, we give a counter example: for $5 \in \mathbb{N}$. and $4 \in \mathbb{N}$

$f(4) = \lfloor 4/2 \rfloor = 2$

$f(5) = \lfloor 5/2 \rfloor = 2$.

→ To show f is onto, we consider $y \in \mathbb{N}$. then $2y \in \mathbb{N}$.

Moreover. $f(2y) = \lfloor 2y/2 \rfloor = \lfloor y \rfloor = y$.

So for any $y \in \mathbb{N}$, there is an $x \in \mathbb{N}$ such that $f(x) = y$.

So. f is onto.

Composition:

P141:

Example 21.

~~Set~~ Let f and g be the functions from ~~the set of integers~~ $\mathbb{Z} \to \mathbb{Z}$ defined by $\begin{cases} f(x) = 2x+3 \\ g(x) = 3x+2. \end{cases}$ what is the composition of f and g? what is $g \circ f$? $f \circ f$? $g \circ g$?

Sol:

$$f \circ g = f(g(x)) = f(3x+2) = 2(3x+2)+3 = 6x+4+3 = 6x+7$$

$$g \circ f = g(f(x)) = g(2x+3) = 3(2x+3)+2 = 6x+9+2 = 6x+11.$$

$$f \circ f = f(f(x)) = f(2x+3) = 2(2x+3)+3 = 4x+6+3 = 4x+9.$$

$$g \circ g = g(g(x)) = g(3x+2) = 3(3x+2)+2 = 9x+6+2 = 9x+8$$

---

* Prove or disprove: Let f, g be functions from $\mathbb{R} \to \mathbb{R}$, then $f \circ g = g \circ f$.

Sol: Disprove by counter example:

$f(x) = 2x+3$

$g(x) = 3x+2.$

$$f \circ g(x) = f(g(x)) = f(3x+2) = \cancel{6x+} \; 2(3x+2)+3 = 6x+7$$

$$g \circ f(x) = g(f(x)) = \cancel{3}\, g(2x+3) = 3(2x+3)+2 = 6x+11$$

$\therefore$ $g \circ f \neq f \circ g$.

Special Functions

1. Prove or disprove each of these statements about the floor and ceiling functions:

a) For all real numbers x,

$$\lfloor \lceil x \rceil \rfloor = \lfloor x \rfloor$$

sol: we disaprove this by giving a counterexample.

Let $x = 1.5$

$\lceil 1.5 \rceil = 2$ so $\lfloor \lceil x \rceil \rfloor = \lfloor \lceil 1.5 \rceil \rfloor = 2$

But $\lfloor 1.5 \rfloor = 1$

$\therefore \lfloor \lceil x \rceil \rfloor \neq \lfloor x \rfloor$

b) $\lfloor x \rfloor = \lceil x \rceil$ iff x is an integer.

proof: "$\Leftarrow$" If x is an integer, then the greatest integer no greater than x is x.

So $\lfloor x \rfloor = x$.

Meenwhile, the least integer no smaller than x is x

so. $\lceil x \rceil = x$.

$\therefore \lfloor x \rfloor = \lceil x \rceil$.

(proof by contradiction) "$\Rightarrow$": assume x is not an integer,

Then $\lceil x \rceil > x$, while $x > \lfloor x \rfloor$

so ~~we~~ get $\lceil x \rceil > \lfloor x \rfloor$ which contradict to $\lfloor x \rfloor = \lceil x \rceil$

So we get a contradiction!

$\therefore$ x is an integer.