CSE 235 Homework 02 Fall 2007

Due 1:30PM, Friday, October 5th, 2007

Name/CSE Login _____

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Instructions Follow instructions *carefully*, failure to do so may result in points being deducted. Clearly label each problem and submit the answers *in order*. It is highly recommended that you typeset your homework using LaTeX or a similar typesetting system. Staple this cover page to the front of a hardcopy of your assignment for easier grading. Late submissions *will not be accepted*. Be sure to show sufficient work to justify your answer(s). If you are asked to prove something, you must give as formal, rigorous, and complete proof as possible. You are to work individually, and all work should be your own. The CSE academic dishonesty policy is in effect (see `http://www.cse.unl.edu/undergrads/academic_integrity.php`).

Partner Policy You may work in pairs, but you must follow these guidelines:

1. You must work on *all* problems *together*. You may not simply partition the work between you.
2. You must use LaTeX and you may divide the typing duties however you wish.
3. You may not discuss problems with other groups or individuals.
4. Hand in only one hard copy under the first author's name.

Problem	Points	Score
A	6	
B	6	
C	6	
D	6	
E	10	
F	8	
G	8	
H	5	
1.5.12	5	
1.5.26	5	
1.6.30	5	
1.6.42	5	
1.7.2	5	
1.7.32	5	
2.1.8	5	
2.1.22	5	
2.2.40	5	
Total	100	

Problem A Let X be a set and $P(x), Q(x)$ be predicates over X. Consider the sets

$$\begin{aligned} Y &= \{y \in X | P(y)\} \\ Z &= \{x \in X | Q(x)\} \end{aligned}$$

Complete the following sentences with propositional formulas involving P, Q.

(a) $Y \subseteq Z$ if and only if

(b) $Y \cap Z = \emptyset$ if and only if

(c) $Y \cup Z = X$ if and only if

Problem B Recall that $A \times B$ denotes the cartesian product of two sets.

(a) Prove or disprove:

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

(b) Prove or disprove:

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Problem C Let $A = \{1, 2, 6\}, B = \{0, 3\}$, and $C = \{1, 3, 9\}$. Find the following

(a) $A \times C$

(b) $A \times B \times C$

(c) $B \times B \times B$

Problem D For each of the following sets, 1) determine whether there is a one-to-one function $f : S \to T$; 2) determine whether there is an onto function $f : S \to T$; and 3) determine if there is a bijective function $f : S \to T$.

(a) $S = \{1, 2, 3\}, T = \{a, b, c\}$

(b) $S = \{a, b\}, T = \{1, 2, 3, 4\}$

(c) $S = \{1, 2, 3, 4\}, T = \{a, b\}$

Problem E Determine whether each of the functions below is onto, and/or one-to-one, prove your answers.

(a) $f : \mathbb{Z} \to \mathbb{Z}, f(x) = x - 1$

(b) $f : \mathbb{R} \to \mathbb{R}, f(x) = -3x^2 + 7$

(c) $f : \mathbb{Z} \to \mathbb{Z}, f(x) = \lceil x \rceil$

(d) $f : \mathbb{R} \to \mathbb{R}, f(x) = \lceil x \rceil$

(e) $f : \mathbb{R} \to \mathbb{Z}, f(x) = \lceil x \rceil$

Problem F Define the following functions (assume that the domains/codomains are defined such that each composition is valid): $f(x) = 2x, g(x) = \frac{x}{(1+x)}, h(x) = \sqrt{x}$. Find

(a) $f \circ g \circ h$

(b) $h \circ g \circ f$

(c) $f \circ f$

(d) $g \circ g$

Problem G Find inverses of the following functions (assume that the domains/codomains are defined such that each function is a bijection).

(a) $f(x) = 5x - 3$

(b) $f(x) = 2x^3$

(c) $f(x) = (2x - 2)^2$

(d) $f(x) = \frac{\sqrt{x}}{6}$

Problem H Use a proof by contraposition to prove that if $x + y \geq 2$ where $x, y \in \mathbb{R}$, then $x \geq 1$ or $y \geq 1$.