

Nebraska	Introduction
Predicate Logic and Quantifiers	Consider the following statements: x > 3,  x = y + 3,  x + y = z
CSE235	x > 3, $x = y + 3$ , $x + y - zThe truth value of these statements has no meaning without specifying the values of x, y, z.$
	However, we <i>can</i> make propositions out of such statements.
	A <i>predicate</i> is a property that is affirmed or denied about the <i>subject</i> (in logic, we say "variable" or "argument") of a <i>statement</i> .
	$\underbrace{x}_{\text{subject}} \underbrace{\text{is greater than 3"}}_{\text{predicate}}$
2/1	Terminology: affirmed = holds = is true; denied = does not hold = is not true.

## Nebraska Propositional Functions

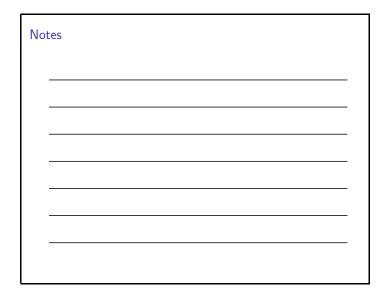
Predicate Logic and Quantifiers

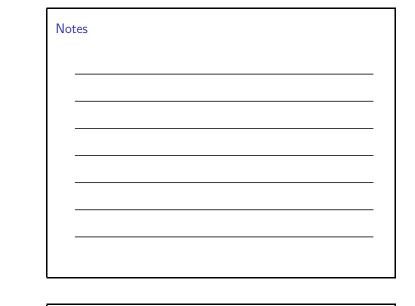
To write in predicate logic:

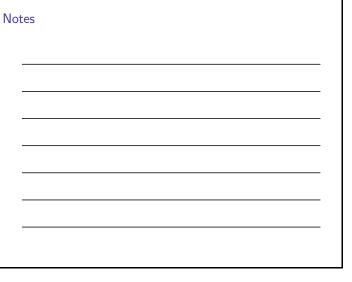
"
$$\underbrace{x}_{\text{subject}}$$
 is greater than 3" predicate

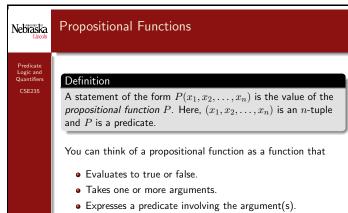
We introduce a (functional) symbol for the predicate, and put the subject as an argument (to the functional symbol): P(x) Examples:

- Father(x): unary predicate
- Brother(*x*,*y*): binary predicate
- Sum(x,y,z): ternary predicate
- P(x,y,z,t): n-ary predicate





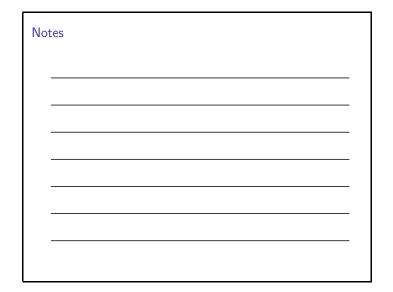


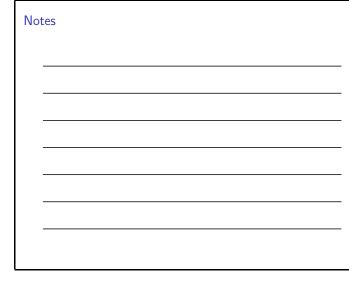


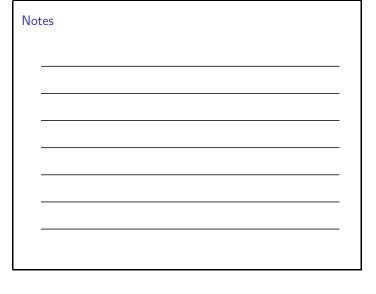
• Becomes a proposition when values are assigned to the arguments.

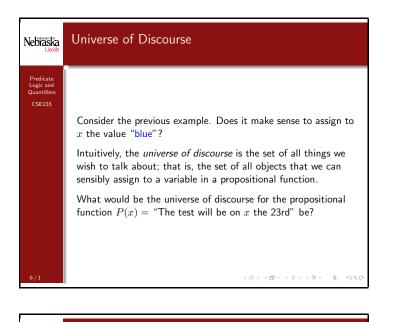
Nebraska Lincoln	Propositional Functions <sub>Example</sub>
Predicate Logic and Quantifiers CSE235	<b>Example</b> Let $Q(x, y, z)$ denote the statement " $x^2 + y^2 = z^{2"}$ . What is the truth value of $Q(3, 4, 5)$ ? What is the truth value of $Q(2, 2, 3)$ ? How many values of $(x, y, z)$ make the predicate true?
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Nebřaska Lincoln	Propositional Functions <sub>Example</sub>
Predicate Logic and Quantifiers CSE235	<b>Example</b> Let $Q(x, y, z)$ denote the statement " $x^2 + y^2 = z^{2"}$ . What is the truth value of $Q(3, 4, 5)$ ? What is the truth value of $Q(2, 2, 3)$ ? How many values of $(x, y, z)$ make the predicate true? Since $3^2 + 4^2 = 25 = 5^2$ , $Q(3, 4, 5)$ is true. Since $2^2 + 2^2 = 8 \neq 3^2 = 9$ , $Q(2, 2, 3)$ is false. There are infinitely many values for $(x, y, z)$ that make this propositional function true—how many right triangles are there?
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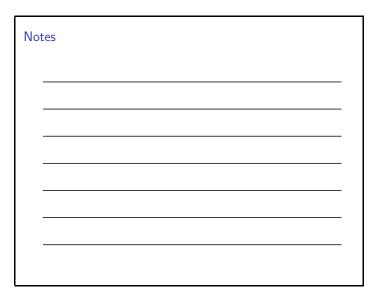


Nebraska	Universe of Discourse Multivariate Functions
Predicate Logic and Quantifiers CSE235	
	Moreover, each variable in an $n$ -tuple may have a different universe of discourse.
	Let $P(r, g, b, c) =$ "The rgb-value of the color $c$ is $(r, g, b)$ ".
	For example, $P(255, 0, 0, red)$ is true, while $P(0, 0, 255, green)$ is false.
	What are the universes of discourse for $(r, g, b, c)$ ?
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A predicate becomes a proposition when we assign it fixed values. However, another way to make a predicate into a proposition is to *quantify* it. That is, the predicate is true (or false) for *all* possible values in the universe of discourse or for *some* value(s) in the universe of discourse.

Such *quantification* can be done with two *quantifiers*: the *universal* quantifier and the *existential* quantifier.



Notes



Predicate Logic and Quantifiers

Nebraska	Universal Quantifier Definition
Predicate Logic and Quantifiers CSE235	<b>Definition</b> The <i>universal quantification</i> of a predicate $P(x)$ is the proposition " $P(x)$ is true for all values of $x$ in the universe of discourse" We use the notation
	$orall x P(x)$ which can be read "for all $x$ " If the universe of discourse is finite, say $\{n_1, n_2, \dots, n_k\}$ , then
	the universal quantifier is simply the conjunction of all elements: $\forall x P(x) \iff P(n_1) \land P(n_2) \land \dots \land P(n_k)$
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Nebraska	Universal Quantifier Example I
Predicate Logic and Quantifiers CSE235	<ul> <li>Let P(x) be the predicate "x must take a discrete mathematics course" and let Q(x) be the predicate "x is a computer science student".</li> <li>The universe of discourse for both P(x) and Q(x) is all UNL students.</li> <li>Express the statement "Every computer science student must take a discrete mathematics course".</li> </ul>
10/1	• Express the statement "Everybody must take a discrete mathematics course or be a computer science student".



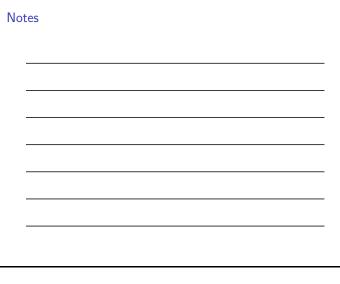
- Let P(x) be the predicate "x must take a discrete mathematics course" and let Q(x) be the predicate "x is a computer science student".
  - $\bullet$  The universe of discourse for both P(x) and Q(x) is all UNL students.
  - Express the statement "Every computer science student must take a discrete mathematics course".

 $\forall x (Q(x) \to P(x))$ 

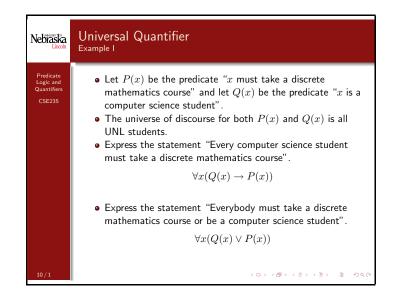
• Express the statement "Everybody must take a discrete mathematics course or be a computer science student".

Notes



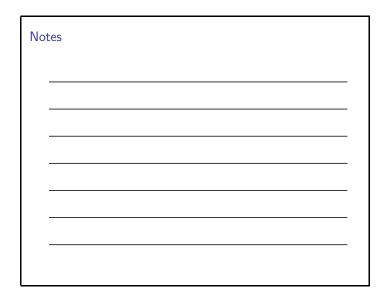


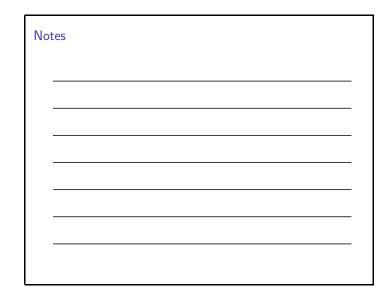
Predicate Logic and Quantifiers

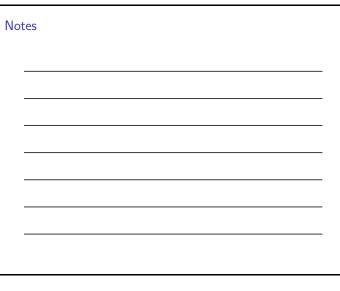


Nebraska	Universal Quantifier <sub>Example 1</sub>
Predicate Logic and Quantifiers CSE235	<ul> <li>Let P(x) be the predicate "x must take a discrete mathematics course" and let Q(x) be the predicate "x is a computer science student".</li> <li>The universe of discourse for both P(x) and Q(x) is all UNL students.</li> <li>Express the statement "Every computer science student must take a discrete mathematics course".</li> <li>∀x(Q(x) → P(x))</li> </ul>
	• Express the statement "Everybody must take a discrete mathematics course or be a computer science student". $\forall x (Q(x) \lor P(x))$
10/1	• Are hetse statements true or false? , , , , , , , , , , , , , , , , , , ,

Nebřaška Lincoln	Universal Quantifier <sub>Example II</sub>
Predicate Logic and Quantifiers CSE235	Express the statement "for every $x$ and for every $y, \; x+y > 10$ "
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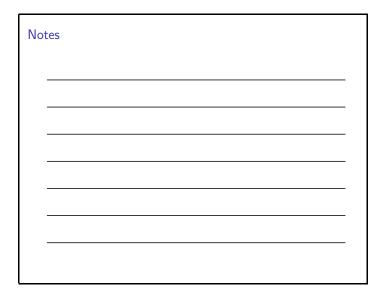


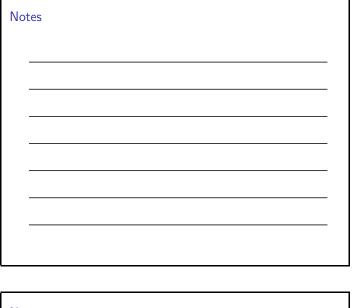


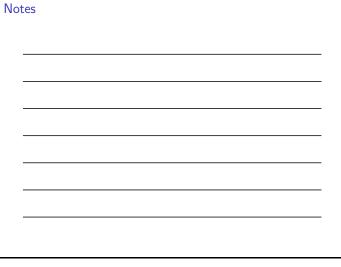
Nebraska Lincoln	Universal Quantifier <sub>Example II</sub>
Predicate Logic and Quantifiers CSE235	Express the statement "for every $x$ and for every $y$ , $x + y > 10$ " Let $P(x, y)$ be the statement $x + y > 10$ where the universe of discourse for $x, y$ is the set of integers.
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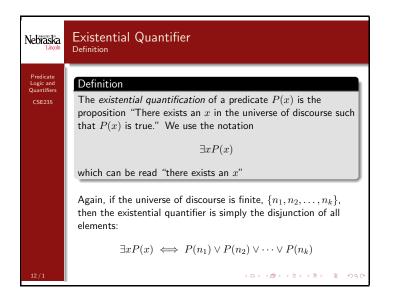
Nebraska	Universal Quantifier Example II
Predicate Logic and Quantifiers CSE235	Express the statement "for every $x$ and for every $y$ , $x + y > 10$ " Let $P(x, y)$ be the statement $x + y > 10$ where the universe of discourse for $x, y$ is the set of integers. Answer: $\forall x \forall y P(x, y)$
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Nebřaska Lincoln	Universal Quantifier <sub>Example II</sub>
Predicate Logic and Quantifiers CSE235	Express the statement "for every $x$ and for every $y,x+y>10"$
	Let $P(x,y)$ be the statement $x+y>10$ where the universe of discourse for $x,y$ is the set of integers.
	Answer: $\forall x \forall y P(x,y)$
	Note that we can also use the shorthand
	orall x, y P(x,y)
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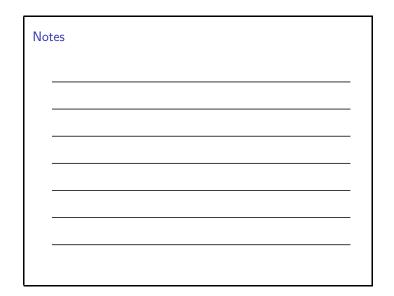


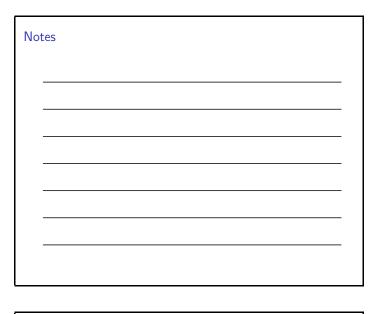


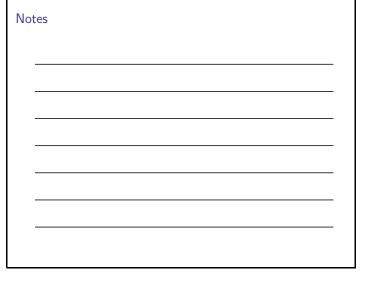


Nebraska	Existential Quantifier <sub>Example I</sub>
Predicate Logic and Quantifiers CSE235	
	Let $P(x,y)$ denote the statement, " $x + y = 5$ ".
	What does the expression,
	$\exists x \exists y P(x,y)$
	mean?
	What universe(s) of discourse make it true?
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Nebraska	Existential Quantifier <sub>Example II</sub>
Predicate Logic and Quantifiers CSE235	Express the statement "there exists a real solution to $ax^2 + bx - c = 0$ "
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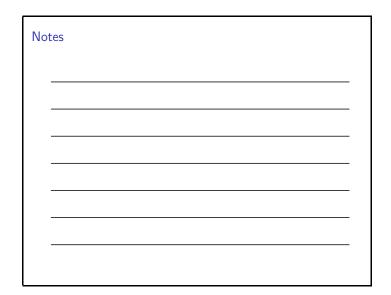


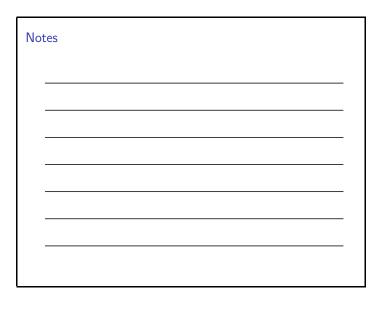


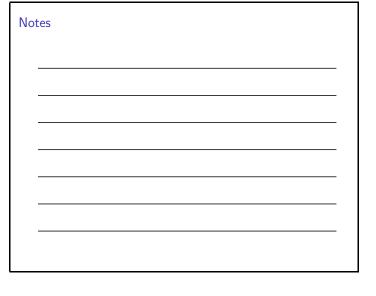
Nebraska	Existential Quantifier <sub>Example II</sub>
Predicate Logic and Quantifiers CSE235	Express the statement "there exists a real solution to $ax^2 + bx - c = 0$ " Let $P(x)$ be the statement $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where the universe of discourse for $x$ is the set of reals. Note here that $a, b, c$ are all fixed constants.
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Nebraska	Existential Quantifier Example II	
Predicate Logic and Quantifiers		
CSE235	Express the statement "there exists a real solution to $ax^2 + bx - c = 0$ "	
	Let $P(x)$ be the statement $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where the universe of discourse for $x$ is the set of reals. Note here that $a, b, c$ are all fixed constants.	
	The statement can thus be expressed as	
	$\exists x P(x)$	
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Nebraska Lincoln	Existential Quantifier Example II Continued
Predicate Logic and Quantifiers CSE235	Question: what is the truth value of $\exists x P(x)$ ?
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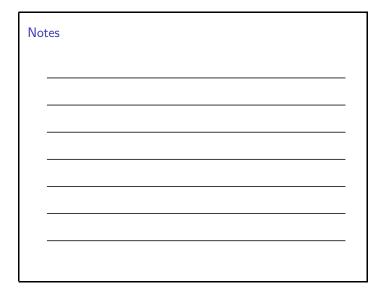




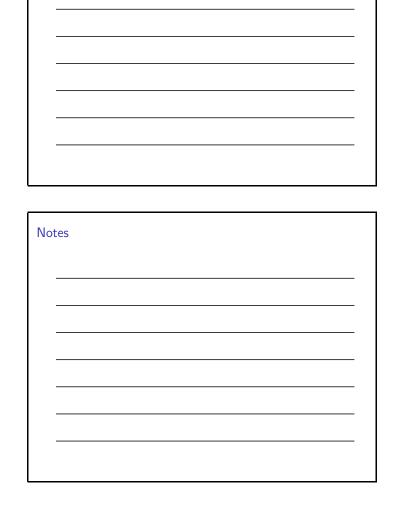
Nebraska Lincoln	Existential Quantifier Example II Continued
Predicate Logic and Quantifiers CSE235	
	Question: what is the truth value of $\exists x P(x)$ ?
	Answer: it is false. For any real numbers such that $b^2 < 4ac$ , there will only be complex solutions, for these cases no such <i>real</i> number $x$ can satisfy the predicate.
	How can we make it so that it <i>is</i> true?
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Nebraska	Existential Quantifier Example II Continued	
Predicate Logic and Quantifiers CSE235		
	Question: what is the truth value of $\exists x P(x)$ ?	
	Answer: it is false. For any real numbers such that $b^2 < 4ac$ , there will only be complex solutions, for these cases no such <i>real</i> number $x$ can satisfy the predicate.	
	How can we make it so that it <i>is</i> true?	
	Answer: change the universe of discourse to the complex numbers, $\mathbb{C}.$	
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Nebraska	Quantifiers Truth Values		
Predicate Logic and Quantifiers CSE235	In general, whe	en are quantified stateme	nts true/false?
	Statement	True When	False When
	$\forall x P(x)$	$\begin{array}{ c c } P(x) \text{ is true for every} \\ x. \end{array}$	There is an $x$ for which $P(x)$ is false.
	$\exists x P(x)$	There is an $x$ for which $P(x)$ is true.	P(x) is false for every $x$ .
		Table: Truth Values of Q	uantifiers
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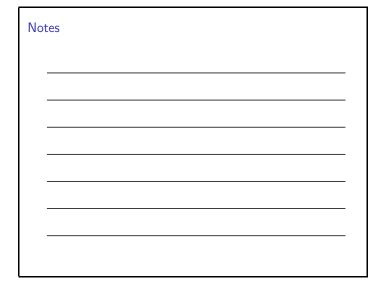
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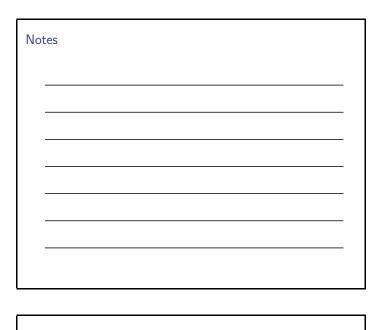


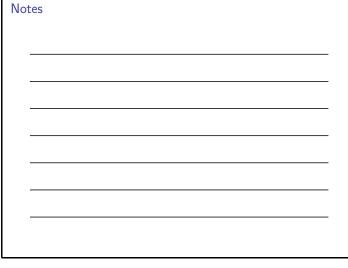
Nebraska Lincoln	Mixing Quantifiers I
Predicate Logic and Quantifiers CSE235	
	Existential and universal quantifiers can be used together to quantify a predicate statement; for example,
	$\forall x \exists y P(x,y)$
	is perfectly valid. However, you must be careful—it must be read left to right.
	For example, $\forall x \exists y P(x, y)$ is not equivalent to $\exists y \forall x P(x, y)$ . Thus, ordering is important.
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Nebraska	Mixing Quantifiers II
Predicate Logic and Quantifiers CSE235	For example: • $\forall x \exists y Loves(x, y)$ : everybody loves somebody • $\exists y \forall x Loves(x, y)$ : There is someone loved by everyone Those expressions do not mean the same thing! Note that $\exists y \forall x P(x, y) \rightarrow \forall x \exists y P(x, y)$ , but the converse does not hold However, you <i>can</i> commute <i>similar</i> quantifiers; $\exists x \exists y P(x, y)$ is equivalent to $\exists y \exists x P(x, y)$ (which is why our shorthand was valid).
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Nebřaska Lincoln	Mixing Quan	tifiers	
Predicate	Statement	True When	False When
Logic and Quantifiers	$\forall x \forall y P(x,y)$	( /0/	There is at least one
CSE235		ery pair $x, y$ .	pair, $x, y$ for which $P(x, y)$ is false.
	$\forall x \exists y P(x,y)$	For every $x$ , there is a	There is an $x$ for
		y for which $P(x, y)$ is true.	which $P(x, y)$ is false for every $y$ .
	$\exists x \forall y P(x,y)$	There is an $x$ for which $P(x, y)$ is true	For every $x$ , there is a $y$ for which $P(x, y)$ is
		for every $y$ .	false.
	$\exists x \exists y P(x,y)$		P(x,y) is false for ev-
		pair $x, y$ for which $P(x, y)$ is true.	ery pair $x, y$ .
	Tab	le: Truth Values of 2-varia	te Quantifiers
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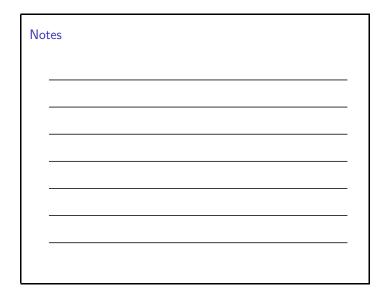




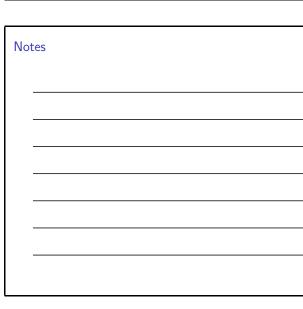
Nebraska Lincoln	Mixing Quantifiers <sub>Example 1</sub>
Predicate Logic and Quantifiers CSE235	Express, in predicate logic, the statement that there are an infinite number of integers.
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Nebřaska	Mixing Quantifiers Example 1
Predicate Logic and Quantifiers CSE235	Express, in predicate logic, the statement that there are an infinite number of integers. Let $P(x, y)$ be the statement that $x < y$ . Let the universe of discourse be the integers, $\mathbb{Z}$ .
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Nebraska	Mixing Quantifiers Example I
Predicate Logic and Quantifiers CSE235	
	Express, in predicate logic, the statement that there are an infinite number of integers.
	Let $P(x,y)$ be the statement that $x < y$ . Let the universe of discourse be the integers, $\mathbb{Z}$ .
	Then the statement can be expressed by the following.
	$\forall x \exists y P(x,y)$
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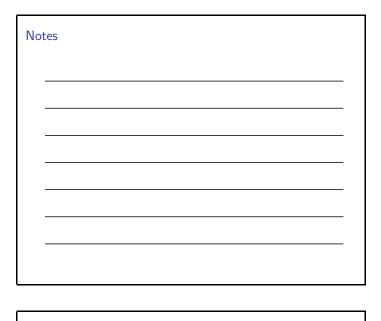
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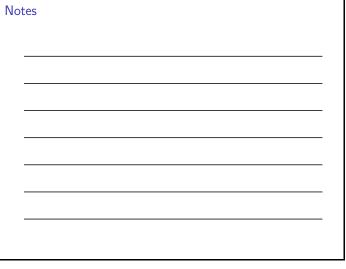
Nebraska	Mixing Quantifiers Example II: More Mathematical Statements
Predicate Logic and Quantifiers CSE235	Express the <i>commutative law of addition</i> for $\mathbb{R}$ .
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Nebraska	Mixing Quantifiers Example II: More Mathematical Statements
Predicate Logic and Quantifiers	
CSE235	Express the <i>commutative law of addition</i> for $\mathbb{R}$ .
	We want to express that for every pair of reals, $x, y$ the following identity holds:
	x + y = y + x
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Nebřaska	Mixing Quantifiers Example II: More Mathematical Statements
Predicate Logic and Quantifiers	
CSE235	Express the <i>commutative law of addition</i> for $\mathbb{R}$ .
	We want to express that for every pair of reals, $x, y$ the following identity holds:
	x + y = y + x
	Then we have the following:
	$\forall x \forall y (x + y = y + x)$
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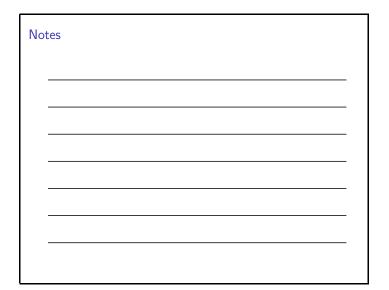


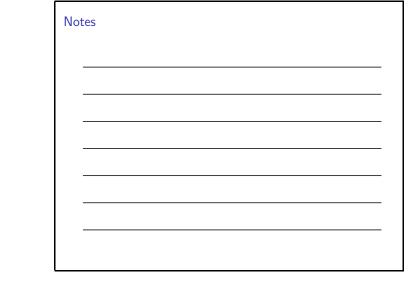


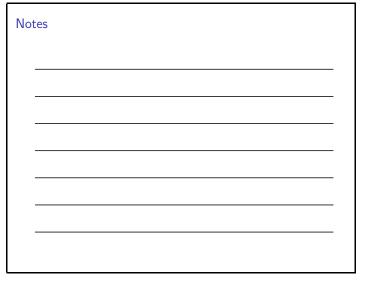
Nebraska Lincoln	Mixing Quantifiers Example II: More Mathematical Statements Continued
Predicate Logic and Quantifiers CSE235	Express the <i>multiplicative inverse law</i> for (nonzero) rationals $\mathbb{Q} \setminus \{0\}$ .
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Nebraska	Mixing Quantifiers Example II: More Mathematical Statements Continued
Predicate Logic and Quantifiers CSE235	Express the multiplicative inverse law for (nonzero) rationals $\mathbb{Q} \setminus \{0\}$ . We want to express that for every real number $x$ , there exists a real number $y$ such that $xy = 1$ .
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Nebraska	Mixing Quantifiers Example II: More Mathematical Statements Continued
Predicate Logic and Quantifiers	
CSE235	Express the multiplicative inverse law for (nonzero) rationals $\mathbb{Q} \setminus \{0\}.$
	We want to express that for every real number $x$ , there exists a real number $y$ such that $xy = 1$ .
	Then we have the following:
	$\forall x \exists y (xy = 1)$
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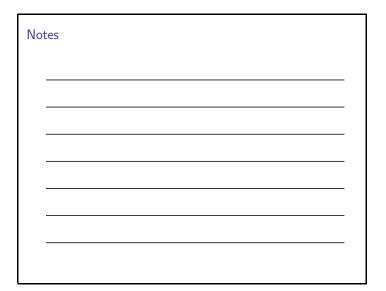


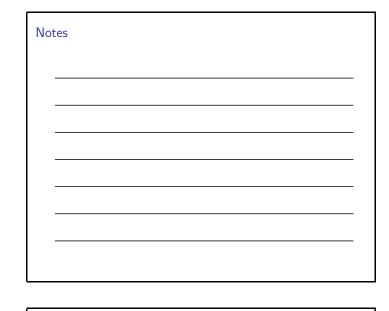


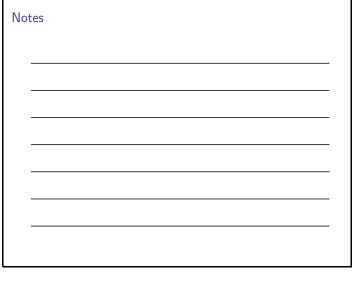
Nebraska Lincoln	Mixing Quantifiers Example II: False Mathematical Statements
Predicate Logic and Quantifiers CSE235	Is commutativity for subtraction valid over the reals?
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Nebraska	Mixing Quantifiers Example II: False Mathematical Statements
Predicate Logic and Quantifiers CSE235	
	Is commutativity for subtraction valid over the reals?
	That is, for all pairs of real numbers $x, y$ does the identity $x - y = y - x$ hold? Express this using quantifiers.
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Nebraska	Mixing Quantifiers Example II: False Mathematical Statements
Predicate Logic and Quantifiers CSE235	
	Is commutativity for subtraction valid over the reals?
	That is, for all pairs of real numbers $x, y$ does the identity $x - y = y - x$ hold? Express this using quantifiers.
	The expression is
	$\forall x \forall y (x - y = y - x)$
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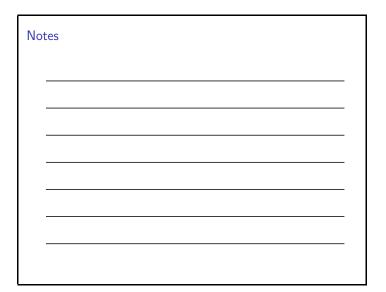




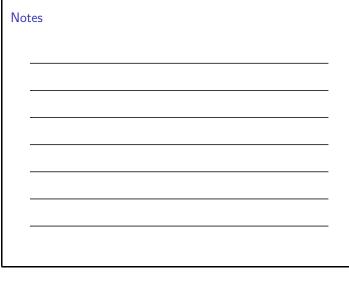
Nebraska	Mixing Quantifiers Example II: False Mathematical Statements Continued
Predicate Logic and Quantifiers CSE235	Is there a multiplicative inverse law over the nonzero integers?
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Nebraska	Mixing Quantifiers Example II: False Mathematical Statements Continued
Predicate Logic and Quantifiers CSE235	
	Is there a multiplicative inverse law over the nonzero integers?
	That is, for every integer $x$ does there exists a $y$ such that $xy=1?$

Nebraska Lincoln	Mixing Quantifiers Example II: False Mathematical Statements Continued
Predicate Logic and Quantifiers CSE235	
	Is there a multiplicative inverse law over the nonzero integers?
	That is, for every integer $x$ does there exists a $y$ such that $xy = 1$ ?
	This is false, since we can find a <i>counter example</i> . Take any integer, say 5 and multiply it with another integer, $y$ . If the statement held, then $5 = 1/y$ , but for any (nonzero) integer $y$ , $ 1/y  \le 1$ .
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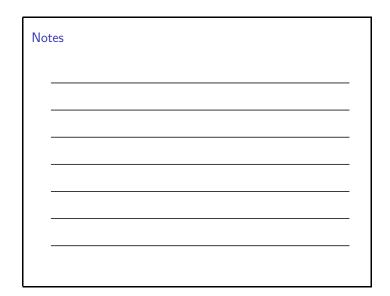


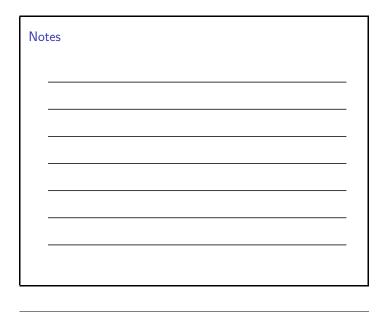


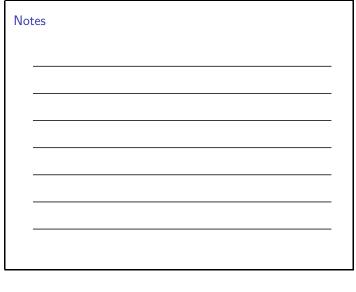
Nebřaška <sub>Lincoln</sub>	Mixing Quantifiers Exercise
Predicate Logic and Quantifiers CSE235	Express the statement "there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x" as a logical expression. Solution:
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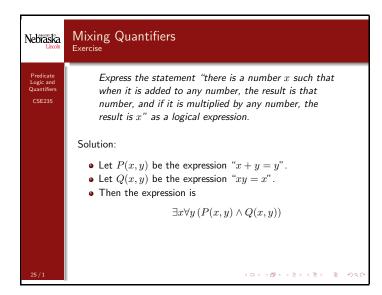
Nebraska	Mixing Quantifiers Exercise
Predicate Logic and Quantifiers CSE235	Express the statement "there is a number $x$ such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is $x$ " as a logical expression.
	Solution: • Let $P(x,y)$ be the expression " $x + y = y$ ".
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Nebraska	Mixing Quantifiers Exercise
Predicate Logic and Quantifiers CSE235	Express the statement "there is a number $x$ such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is $x''$ as a logical expression.
	Solution: • Let $P(x, y)$ be the expression " $x + y = y$ ". • Let $Q(x, y)$ be the expression " $xy = x$ ".
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Nebraska	Mixing Quantifiers Exercise
Predicate Logic and Quantifiers CSE235	Express the statement "there is a number $x$ such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is $x^n$ as a logical expression.
	Solution:
	<ul> <li>Let P(x, y) be the expression "x + y = y".</li> <li>Let Q(x, y) be the expression "xy = x".</li> <li>Then the expression is</li> </ul>
	$\exists x \forall y \left( P(x,y) \land Q(x,y) \right)$
	<ul> <li>Over what universe(s) of discourse does this statement hold?</li> </ul>
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## Nebraska Lixon Mixing Quantifiers

Express the statement "there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x" as a logical expression.

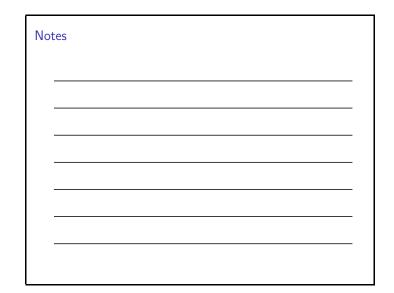
## Solution:

Predicate Logic and Quantifiers

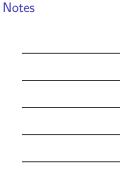
- Let P(x,y) be the expression "x + y = y".
- Let Q(x,y) be the expression "xy = x".
- Then the expression is

 $\exists x \forall y \left( P(x,y) \land Q(x,y) \right)$ 

- $\bullet\,$  Over what universe(s) of discourse does this statement hold?
- This is the *additive identity law* and holds for  $\mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathbb{Q}$ but does not hold for  $\mathbb{Z}^+$ .



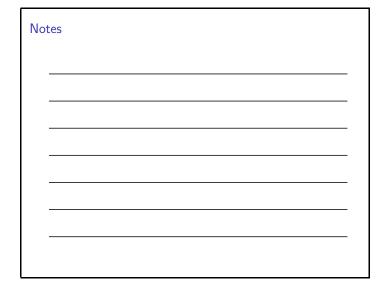


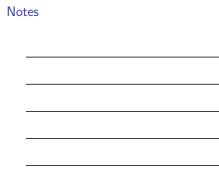


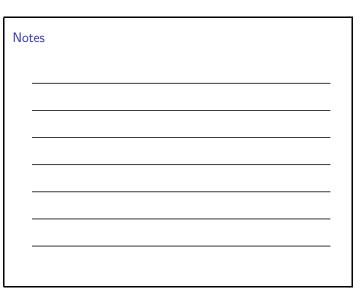
Nebraska Lincoln	Binding Variables I
Predicate Logic and Quantifiers CSE235	When a quantifier is used on a variable $x$ , we say that $x$ is <i>bound</i> . If no quantifier is used on a variable in a predicate statement, it is called <i>free</i> .
	<b>Example</b> In the expression $\exists x \forall y P(x, y)$ both $x$ and $y$ are bound. In the expression $\forall x P(x, y), x$ is bound, but $y$ is free.
	A statement is called a <i>well-formed formula</i> , when all variables are properly quantified.
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Nebraska	Binding Variables II
Predicate Logic and Quantifiers CSE235	
C3E235	The set of all variables bound by a common quantifier is the <i>scope</i> of that quantifier.
	<b>Example</b> In the expression $\exists x, y \forall z P(x, y, z, c)$ the scope of the existential quantifier is $\{x, y\}$ , the scope of the universal quantifier is just $z$ and $c$ has no scope since it is free.
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Nebraska	Negation
Predicate Logic and Quantifiers CSE235	Just as we can use negation with propositions, we can use them with quantified expressions. Lemma
	Let $P(x)$ be a predicate. Then the following hold.
	$\neg \forall x P(x) \equiv \exists x \neg P(x)$
	$\neg \exists x P(x) \equiv \forall x \neg P(x)$
	This is essentially a quantified version of De Morgan's Law (in fact if the universe of discourse is finite, it is <i>exactly</i> De Morgan's law).
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Nebraska Lincoln	Negation Truth Values
Predicate Logic and Quantifiers CSE235	
	Statement True When False When
	$ \begin{array}{ c c c c c } \hline \neg \exists x P(x) \equiv & \mbox{For every } x, \ P(x) \ \mbox{is} & \mbox{There is an } x \ \mbox{for} \\ \forall x \neg P(x) & \mbox{false.} & \mbox{which } P(x) \ \mbox{is true.} \end{array} $
	$ \begin{array}{c c} \neg \forall x P(x) \equiv & \text{There is an } x \text{ for } P(x) \text{ is true for every} \\ \exists x \neg P(x) & \text{which } P(x) \text{ is false.} & x. \end{array} $
	Table: Truth Values of Negated Quantifiers
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Nebraska	Prolog
Predicate Logic and Quantifiers CSE235	Prolog (Programming in Logic) is a programming language based on (a restricted form of) Predicate Calculus. It was developped by the logicians of the artificial intelligence community for symbolic reasoning.
30/1	<ul> <li>Prolog allows the user to express facts and rules</li> <li>Facts are proposational functions: student(juana), enrolled(juana,cse235), instructor(patel,cse235), etc.</li> <li>Rules are implications with conjunctions: teaches(X,Y) :- instructor(X,Z), enrolled(Y,Z)</li> <li>Prolog answers queries such as: ?enrolled(juana,cse478) ?enrolled(X,cse478)</li> <li>?teaches(X,juana)</li> <li>by binding variables and doing theorem proving (i.e., applying inference rules) as we will see in Section 1.5.</li> </ul>

## Nebraska English into Logic

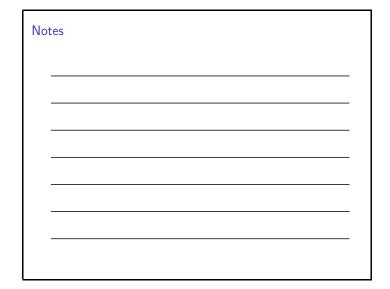
Predicate Logic and Quantifiers

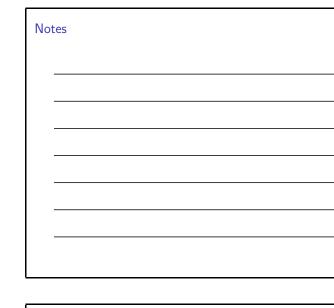
Logic is more precise than English.

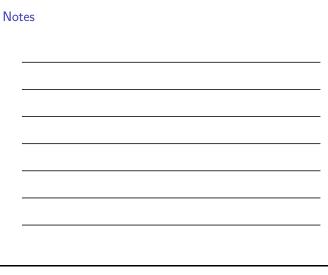
Transcribing English to Logic and vice versa can be tricky.

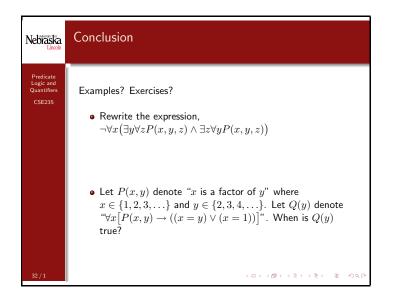
When writing statements with quantifiers, *usually* the correct meaning is conveyed with the following combinations:

- Use  $\forall$  with  $\Rightarrow$ Example:  $\forall xLion(x) \Rightarrow Fierce(x)$  $\forall xLion(x) \land Fierce(x)$  means "everyone is a lion and everyone is fierce"
- Use  $\exists$  with  $\land$ Example:  $\exists xLion(x) \land Drinks(x, coffee)$ : holds when you have at least one lion that drinks coffee  $\exists xLion(x) \Rightarrow Drinks(x, coffee)$  holds when you have people even though no lion drinks coffee.









Nebraska	Conclusion
Predicate Logic and Quantifiers CSE235	Examples? Exercises? • Rewrite the expression, $\neg \forall x (\exists y \forall z P(x, y, z) \land \exists z \forall y P(x, y, z))$ • Answer: Use the negated quantifiers and De Morgan's law. $\exists x (\forall y \exists z \neg P(x, y, z) \lor \forall z \exists y \neg P(x, y, z))$ • Let $P(x, y)$ denote "x is a factor of y" where $x \in \{1, 2, 3,\}$ and $y \in \{2, 3, 4,\}$ . Let $Q(y)$ denote " $\forall x [P(x, y) \rightarrow ((x = y) \lor (x = 1))]$ ". When is $Q(y)$ true?
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