Discrete Mathematics: Introduction

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Computer Science & Engineering 235
Introduction to Discrete Mathematics
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- Roll
- Syllabus
- Lectures: M/W/F 1:30 – 2:20 (Avery 109)
- Recitations: Tuesdays 5:30 – 6:20 (Avery 108)
- Office hours:
  - Instructor: M/W 2:30 – 3:30 (Avery 123B)
  - TA: Chris Bourke: 1:00 – 2:00 (Wed in Avery 123C and Thu in Avery 13A)
- Must have cse account
- Must use webhandin
- Bonus points: report bugs

Why Discrete Mathematics? I

You have to.

Computer Science is *not* programming.

It is not even Software Engineering.

> *"Computer Science is no more about computers than astronomy is about telescopes." –Edsger Dijkstra*

Computer Science is *problem solving*.

Why Discrete Mathematics? II

Mathematics is at the heart of problem solving.

Often, even *defining* a problem requires a level of mathematical rigor.

Competent use and analysis of models/data structures/algorithms requires a solid foundation in mathematics.

Justification for why a particular way of solving a problem is *correct* or *efficient* (i.e., better than another way) requires analysis within a well defined mathematical model.

Why Discrete Mathematics? III

Abstract thinking is necessary to applying knowledge.

Rarely will you encounter a problem in an abstract setting (your boss is not going to ask you to solve MST). Rather, it is up to you to determine the proper model of such a problem.

Scenario I

A limo company has hired you (or your company) to write a computer program to automate the following tasks for a large event.

Task 1 – In the first scenario, businesses request limos and drivers for a fixed period of time (specifying a start-date/time and end-date/time) and charged a flat rate. The program should be able to generate a schedule so that the maximum number of customers can be accommodated.

Scenario II

Task 2 – In the second scenario, the limo service is considering allowing customers to *bid* on a driver (so that the highest bidder gets a limo/driver when there aren't enough available). The program should thus make a schedule a feasible (i.e., no limo can handle two customers at the same time) while at the same time, maximizing the profit by selecting the highest *overall* bids.

Scenario III

Task 3 – In a third scenario, a customer is allowed to specify a *set* of various times and bid an amount for the entire event. A driver must choose to accept the entire set of times or reject it all. The scheduler must still maximize the profit.

Scenario

What's your solution?

How can you *model* such scenarios?

How can you develop algorithms for these scenarios?

How can you justify that that they work? That they actually guarantee an optimal (i.e., maximized profit) solution?

Scenario

The fundamentals that this course will teach you are the foundations that you will use to eventually solve these problems.

The first scenario is easily (i.e., efficiently) solved by a *greedy algorithm*.

The second scenario is also efficiently solvable, but by a more involved technique, *dynamic programming*.

The last scenario is not efficiently solvable (it is NP-hard) by any known technique. It is believed that to guarantee an optimal solution, one needs to look at all (exponentially many) possibilities.

A *set* is a collection of similar objects. We denote a set using brackets. For example,

$$S = \{s_1, s_2, s_3, \ldots, s_n\}$$

is a finite set and

$$S = \{s_1, s_2, s_3, \ldots\}$$

is an infinite set.

We denote that an object is an *element* of a set by the notation,

$$s_1 \in S$$

read "s_1 (is) in S" (or we can write $s_1 \notin S$ for "s_1 (is) *not* in S")

Fundamentals

Notation

You should at least be familiar with the sets of integers, rationals and reals.

- We denote the set of *natural numbers* as

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

- We denote the set of *integers* as

$$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \ldots\}$$

- We denote the set of *rational* numbers as

$$\mathbb{Q} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z} \right\}$$

- We denote the set of *reals* as

$$\mathbb{R} = \{x \mid x \text{ is a decimal number}\}$$

Algebra I

Definition

Let $a, b \in \mathbb{Z}$ with $b \neq 0$. we say that b *divides* a if and only if

$$a = qb$$

for some integer q. We will use the notation

$$b \mid a$$

Algebra II

Example

2 divides 64 since 64 $= 32 \times 2$

$$2 \mid 32$$

3 divides 27 since 27 $= 9 \times 3$

$$3 \mid 27$$

However, 2 does not divide 27 since there is no integer q such that

$$27 = 2q$$

In this case, we write $2 \nmid 27$

Topics

Topic	Sections
Propositional Logic	1.1 – 1.2
Predicate Logic	1.3 – 1.4
Proofs	1.5 – 1.6
Sets	2.1 – 2.2
Functions	2.3
Relations	8.1, 8.3 – 8.6
Algorithms	3.1 – 3.3
Number Theory	3.4 – 3.7
Induction	4.1 – 4.2
Counting	5.1 – 5.2
Combinatorics	5.3 – 5.5
Recursion	7.1 – 7.2
PIE	7.5
Graphs	9.1 – 9.5
Trees	10.1 – 10.3