**Title:** Search Orders in CSPs
**Context:** Finite domains, binary constraints
**Reading:** Foundations of Constraint Satisfaction, Edward Tsang, Chapter 6 (*required*)
Dual Viewpoint Heuristics for Binary Constraint Satisfaction Problems, Geelen 92 (*Recommended*)

**Foundations of Constraint Satisfaction**
**CSCE421/821, Fall 2003**
`www.cse.unl.edu/~choueiry/F03-421-821/`

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## Motivation for ordering heuristics

- In BT, less of undoing-labels needs to be done under some orderings than others
- In lookahead:
  - failure could be detected earlier under some orderings than others
  - larger portions of the solution space can be pruned off under some orderings than others
- When searching for one solution, value ordering may speed up search as branches that have a better chance to reach a solution are explored first

## Heuristics

- **Variable ordering**: Fail first principal (FFP)
  $\longrightarrow$ This terminology is historic and currently considered incorrect. Better use: most constraining first.
- **Value ordering**: 'get quickly to a solution'

**Applying ordering heuristics**: static, dynamic?







## Variable ordering: Fail first principal (FFP)

- Least domain (LD)
- Minimal width ordering (MWO)
- Minimal bandwidth ordering (MBO)
- Maximal cardinality ordering (MCO)
- Minimal ratio domain size over degree (MDD)

**Value ordering**: 'get quickly to a solution'

- Min-conflict heuristic: orders values according to the conflicts in which they are involved with the future variables
- Cruciality [Keng and Yu'89]
- Promise [Geelen'94], etc.



**Fail first principal** (FFP)

Aims at recognizing dead-ends as soon as possible so that search effort can be saved.

Several FFP measures:

- smallest domain size first (least domain, LD)
- smallest of ratio of domain size to degree of node
- etc.

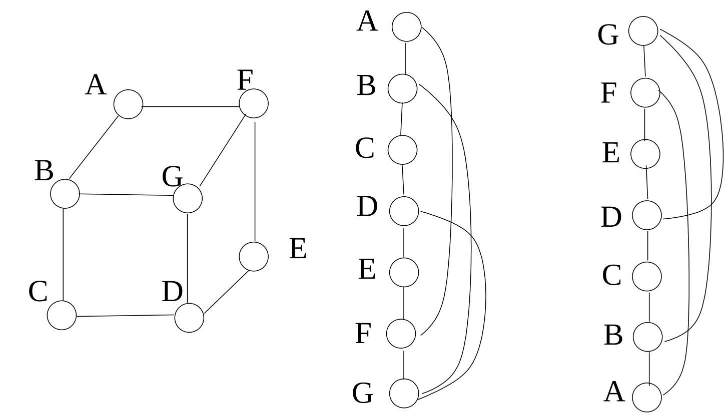
Simple, cheap and effective
Suitable for both static and dynamic ordering

## Width of graph: A graph-theoretic criterion

- Constraint graph
- Ordering of the nodes *how many possible orderings?*
- Width of an ordering
- Width of the graph (independent of the ordering)



## Minimal width ordering




Reduces the chance of backtracking: variables that have more unassigned values depending on them are labeled first. Variables at the front of the ordering are in general more constrained..

Finding minimum width ordering: $\mathcal{O}(n^2)$.

*Freuder'82*

**Procedure** Width of a graph G:
Remove from the graph all nodes not connected to any others
Set $k \leftarrow 0$
Do while there are nodes left in the graph:
    Set $k \leftarrow (k+1)$
    Do while there are nodes not connected to more than $k$ others:
        Remove such nodes from the graph, along
        with any edges connected to them.
Return $k$
**end**

The minimal width ordering of the nodes is obtained by taking the nodes in the reverse order than they were removed.







## Minimal bandwidth ordering

- Localizes/confines backtracking:
  the smaller the bandwidth, the sooner one could backtrack to relevant decisions
- Finding minimum bandwidth ordering is NP-hard :—(
- Is there an ordering of a *given* bandwidth $k$?
  $\rightarrow \mathcal{O}(n^{k+1})$ (polynomial)

**Notes:**
(Zabih'90) finds a correlation between bandwidth and the possibility to decompose the CSP (conjunctive decomposition)

Others (Amir?) consider tree-witdth.

## Maximum Cardinality Ordering

An approximation of min. width ordering, $\mathcal{O}(n)$

- Choose a node arbitrarily.
- Among the remaining nodes, choose the one that is connected to the maximum number of already chosen nodes,
  break ties arbitrarily,
- Repeat...
- Reverse the final order







## **Ordering heuristics**: how, when?

**How:**

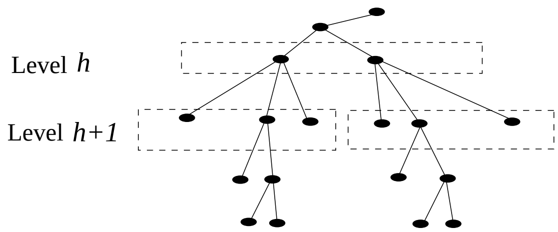
- Static variable, value ordering
- Dynamic variable (static value)
- Dynamic variable, dynamic value (dynamic vvp)

**When:**

- Finding one solution
- Finding all solutions

## Search and ordering heuristics *Choueiry & Beckwith, 2001*

At a given level of the search tree, we encounter:



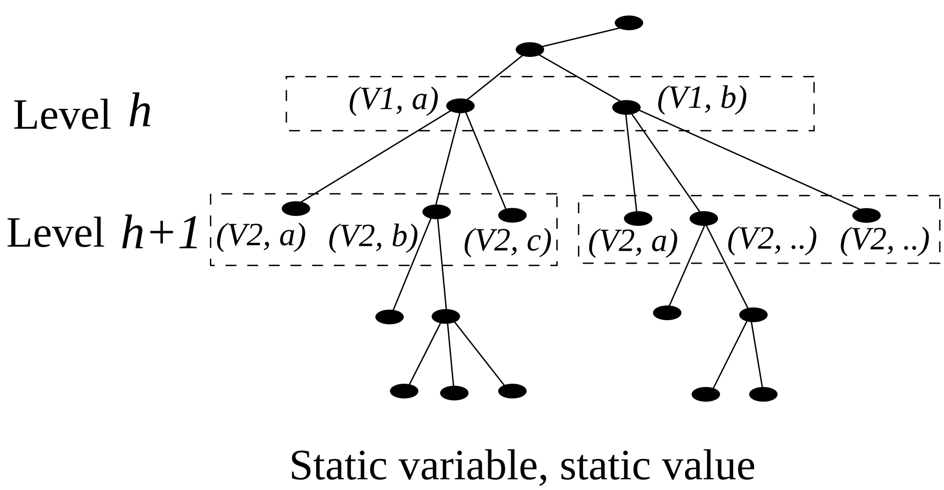

- *static ordering*: vvp's pertaining to the same variable across the whole level
- *dynamic variable, static value ordering*: vvp's pertaining to the same variable for nodes with a common parent, but possibly to different variables for nodes with different parents
- *dynamic vvp ordering*: vvp's pertaining to different variables



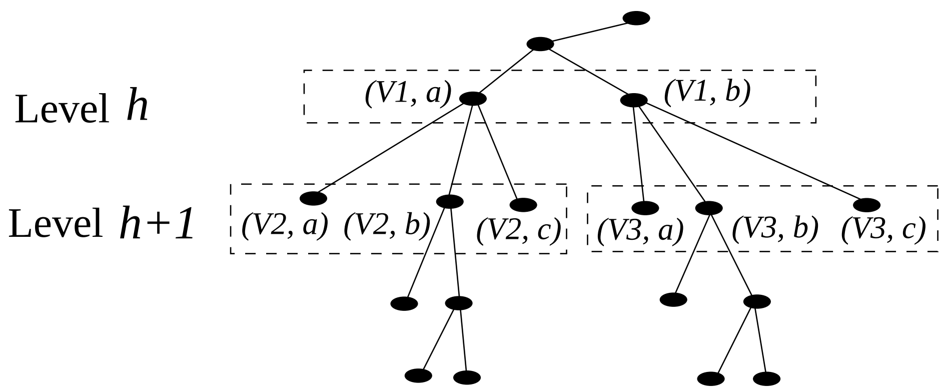




## Static variable ordering, static value ordering



## Dynamic variable ordering, static value ordering



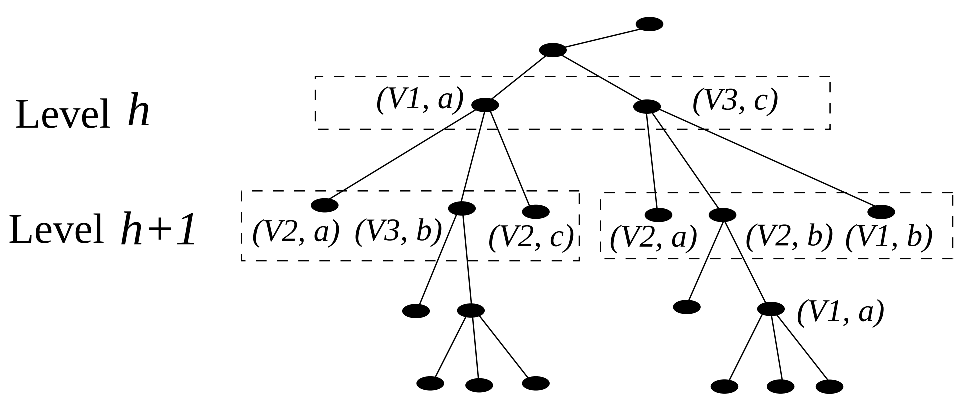

Dynamic variable, static value







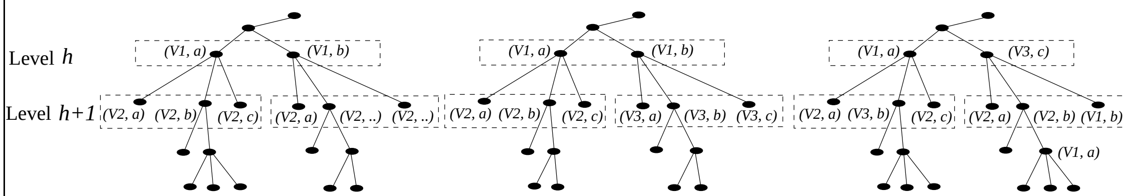
## Dynamic vvp ordering




Dynamic variable-value pair

## **Ordering heuristics**: careful in search

*Choueiry & Beckwith, 2001*




Static variable, static value    Dynamic variable, static value    Dynamic variable-value pair