Fall 2002 **B.Y. Choueiry**

CSCE 421/821: Foundations of Constraint Processing

Homework 2

Assigned: Tuesday, Sep 24, 2002

Due: Tuesday, Oct 8, 2002

Total value: 108 points

Notes: This homework must be done individually. Please avoid discussing it with classmmates, TA or instructor. *If you receive help from anyone, you must clearly acknowledge it.* Always acknowledge sources of information (URL, book, class notes, etc.). Please inform instructor quickly about typos or other errors.

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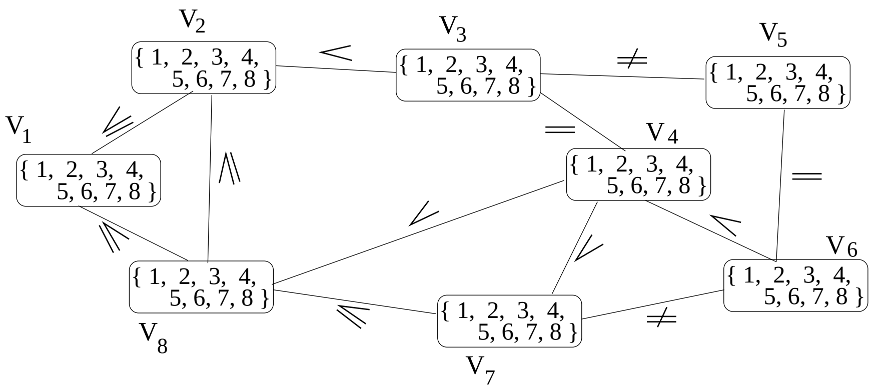
1. **Temporal network** **(Total 20 points)**

   Courtesy of Rina Dechter

   Consider the following constraint satisfaction problem with 8 variables and 8 constraints. Find the equivalent arc- and path-consistent problem. (each 15 points)



2. **Relational algebra** **(Total 8 points)**

Given the three variables X_1, X_2, and X_3, each with the domain $\{a, b, c\}$, and the ternary relation $R_{X_1,X_2,X_3} = \{(a, b, c), (c, b, a), (a, c, a)\}$ of allowed tuples.

Compute:

(a) $R_{X_1,X_2} = \pi_{X_1,X_2} R_{X_1,X_2,X_3}$ (1 point)

(b) $R_{X_1,X_3} = \pi_{X_1,X_3} R_{X_1,X_2,X_3}$ (1 point)

(c) $R_{X_2,X_3} = \pi_{X_2,X_3} R_{X_1,X_2,X_3}$ (1 point)

(d) $R'_{X_1,X_2,X_3} = R_{X_1,X_2} \bowtie R_{X_1,X_3} \bowtie R_{X_2,X_3}$. (2 points)

(e) Compare R_{X_1,X_2,X_3} and R'_{X_1,X_2,X_3}. (1 point)

(f) Can you draw any conclusions on the projection technique? (2 points)

3. **Zebra problem** **(Total 80 points)**

Consider the following puzzle, known as the *Zebra problem*.

Formulation: There are five *houses*, each of a different *color* and inhabited by men of different *nationalities*, with different *pets*, *drinks* and *cigarettes*. Consider the following facts:

(a) The Englishman lives in the red house
(b) The Spaniard has a dog
(c) Coffee is drunk in the green house
(d) The Ukrainian drinks tea
(e) The green house is immediately to the right of the ivory house
(f) The snail owner smokes Old-Gold
(g) Kools are smoked in the yellow house
(h) Milk is drunk in the middle house
(i) The Norwegian lives in the first house on the left
(j) The Chesterfield smoker lives next door to the fox owner
(k) The Kools smoker lives next door to where the horse is kept
(l) The Lucky Strikes smoker drinks orange juice
(m) The Japanese smokes Parliaments
(n) The Norwegian lives next to the blue house

Query: determine who owns the zebra and who drinks water.

Our goal is to model this problem as a CSP, then test some consistency checking algorithms on this CSP. In the following homework, you will have to encode backtrack search algorithms for this CSP. *Please make sure that you keep your code and protect your files.*

(a) **(20 points)** Articulate the variables, the values and the constraints in extension. Here are some hints:
   - Make a variable for each color, drink, *etc.*, so that you have 25 variables.
   - The domain of each variable should be *house positions*: $\{1, 2, 3, 4, 5\}$. For example, assigning the value 2 to the variable horse means that the horse owner lives in the second house.
   - Express all the above enumerated conditions as either unary or binary constraints.

(b) **(10 points)** Implement, in the language or your choice, the data structures for representing the variables, their domain and the constraints. Print out the CSP variables and domains.

(c) **(5 points)** Write a function that takes a variable-value pair and a constraint and checks whether the vvp is supported by the other variable in the (binary) constraint.

(d) **(5 points)** Write a function that takes a variable and returns its 'neighboring' variables, i.e. variables that share a constraint with it.

(e) **(2 points)** Write a function that determines the degree of a variables.

(f) **(10 points)** Implement and run node consistency.
   - Print out and turn in the resulting CSP's variables and domains.
   - Report the number of constraint checks performed.
   - What is the size of the resulting CSP?

(g) **(20 points)** Implement and run arc-consistency (AC-3).
   - Print out and turn in the resulting CSP's variables and domains.
   - Report the number of constraint checks performed.
   - What is the size of the resulting CSP?

(h) Write functions that sort variables by:
   i. **(3 points)** increasing domain size
   ii. **(2 points)** increasing node degree
   iii. **(3 points)** increasing ratio of domain size over degree

   breaking ties lexicographically. These orders will be used for ordering heuristics during search in the following homework.

General indications:

- Declare a data structure to store a given instance of a CSP. For example, this can be a record with the following attributes: a unique problem identifier, a list (or hash-table) of the variables in the CSP, a list (or hash-table) of the constraints in the CSP.

- Declare data structures to represent the CSP variables and constraints. For instance, the data structure for representing a CSP variable can be a record that has a unique identifier, a name of the variable, its initial domain, its current domain (filtered), its assigned value, a set of pointers to the constraints that apply to it. The data structure for the constraint can be a record that has a unique identifier, a set of pointers to the variables in the scope of the constraint, and constraint definition that list all variable-value pairs that are consistent (constraint defined by extension), and/or a method that determined whether two variable-value pairs are consistent (constraint defined by intension).
- Test and debug your algorithm on an a small n-queen problem or the scheduling problem of Homework 1 before you test it on the Zebra problem.