**Title:** Dual Viewpoint Heuristics for Binary Constraint Satisfaction Problems

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**Foundations of Constraint Satisfaction**

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## Contributions

I- New heuristics for variable & value selection

II- Double-viewpoint strategy
(common in scheduling: job vs. resource-centered perspective)
[Sadeh '91]

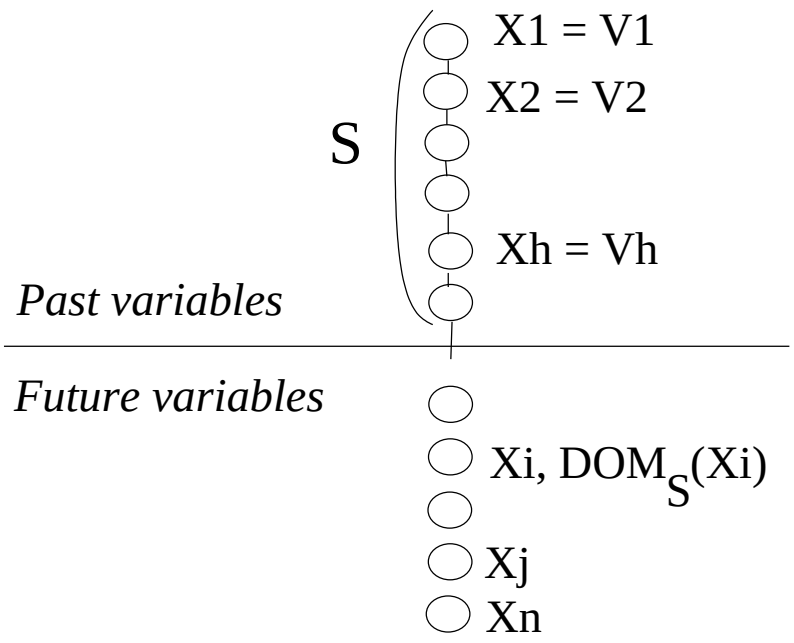
III- Validation on the $n$-Queen problem

## Assumptions

- Binary constraints, finite domains
- Seeking **one** solution (relevant for value ordering)
- Using backtracking (BT) and forward-checking (FC)

# I- Goal of Var/Val orderings in BT

- **avoid constraint violation**
  → select values that do not cause constraint violation
  → most promising value first
- **discover constraint violation quickly**
  → select variables that do not delay constraint violation
  → most constrained variable first (fail first principle)




After forward-checking given $S$ $\begin{cases} \mathrm{DOM}_S(X_i) \\ \vdots \\ \mathrm{DOM}_S(X_j) \\ \mathrm{DOM}_S(X_n) \end{cases}$

For $X_i = V_i$, we can separate $\mathrm{DOM}_S(X_j)$ into two sets in $\mathcal{O}(a)$:

- values consistent with $X_i = V_i$
  $\rightarrow$ of size $\mathrm{LEFT}_S(X_j \mid X_i = V_i)$

- values inconsistent with $X_i = V_i$
  $\rightarrow$ of size $\mathrm{LOST}_S(X_j \mid X_i = V_i)$

## Value selection for $X_i$

Choose the least constraining value $V_i$

1. Minimize $X_j$ future variables

$$\text{Cost}_S(X_i = V_i) = \sum_{X_{j \neq i}} \text{LOST}_S(X_j \mid X_i = V_i)$$

2. Minimize $X_j$ future variables

$$\text{Cruciality}_S(X_i = V_i) = \sum_{X_{j \neq i}} \frac{\text{LOST}_S(X_j \mid X_i = V_i)}{|\text{DOM}_S(X_j)|}$$

3. Maximize $X_j$ future variables

$$\text{Promise}_S(X_i = V_i) = \prod_{X_{j \neq i}} \text{LEFT}_S(X_j \mid X_i = V_i)$$

$\rightarrow$ number of assignments that $X_i = V_i$ and can be done such that no constraint on $X_i$ is violated

(3) is more discriminating $(\text{LEFT}_S(X_j \mid X_i = V_i) = 0)$

# Value selection: example

- Minimize $X_j$ future variables

$$\text{Cost}_S(X_i = V_i) = \sum_{X_{j \neq i}} \text{LOST}_S(X_j \mid X_i = V_i)$$

| | | | | |
|---|---|---|---|---|
| X1 | 6 | 6 | 6 | 6 |
| X2 | 6 | 8 | 8 | 6 |
| X3 | 6 | 8 | 8 | 6 |
| X4 | 6 | 6 | 6 | 6 |

- Maximize $X_j$ future variables

$$\text{Promise}_S(X_i = V_i) = \prod_{X_{j \neq i}} \text{LEFT}_S(X_j \mid X_i = V_i)$$

| | | | | |
|---|---|---|---|---|
| X1 | 8 | 6 | 6 | 8 |
| X2 | 8 | 2 | 2 | 8 |
| X3 | 8 | 2 | 2 | 8 |
| X4 | 8 | 6 | 6 | 8 |

## Variable selection

Choose the most constrained variable (FFP) $X_i$

1. Least domain (LD)

2. Maximize $V_i \in \mathrm{DOM}_S(X_i)$

   $\mathrm{Criticality}_S(X_i) =$

   $$\prod_{V_i} \frac{1}{(1+|\mathrm{DOM}_S(X_i)|+\mathrm{Cruciality}_S(X_i=Vi))}$$

3. Minimize $V_i \in \mathrm{DOM}_S(X_i)$

   $$\mathrm{Promise}_S(X_i) = \sum_{V_i} \mathrm{Promise}_S(X_i = V_i)$$

   $\rightarrow$ number of assignments that can be done such that no constraint on $X_i$ is violated

## Variable selection: example

Minimize: $V_i \in \mathrm{DOM}_S(X_i)$

$$\mathrm{Promise}_S(X_i) = \sum_{V_i} \mathrm{Promise}_S(X_i = V_i)$$

| | | | | | |
|---|---|---|---|---|---|
| X1 | 8 | 6 | 6 | 8 | 28 |
| X2 | 8 | 2 | 2 | 8 | 20 |
| X3 | 8 | 2 | 2 | 8 | 20 |
| X4 | 8 | 6 | 6 | 8 | 28 |

$X_1, X_4$ promise 28 solutions
$X_2, X_3$ promise 20 solutions, more constraining

Start with $X_2$ or $X_3$ (more constraining) and, choose columns 1 or 4 (more promising)

# Summary

Most promising value:

(1) Minimum cost

(2) Minimum cruciality *[Keng & Yun, 89]*

(3) Maximum promise *[Geelen'92]*

Most constrained variable:

- Least domain (LD)

(4) Maximum criticality *[Keng & Yun, 89]*

(5) Minimum (false) promise *[Geelen'92]*

$\longrightarrow$ Dynamic variable/value orderings

# Algorithms

| Identifier | Choice of Var | Choice of val |
|---|---|---|
| LD+1 | least domain | Minimum cost |
| LD+2 | – | Min. cruciality |
| LD+3 | – | Max promise |
| FE+2/4 | Max. critical | Min. cruciality |
| FE+3/5 | Min. promise | Max. promise |

LD: time $\mathcal{O}(na^2)$, space $\mathcal{O}(na)$

FE: time $\mathcal{O}(n^2a^2)$, space $\mathcal{O}(na)$

Implementation hack: domino effect, saves computations.

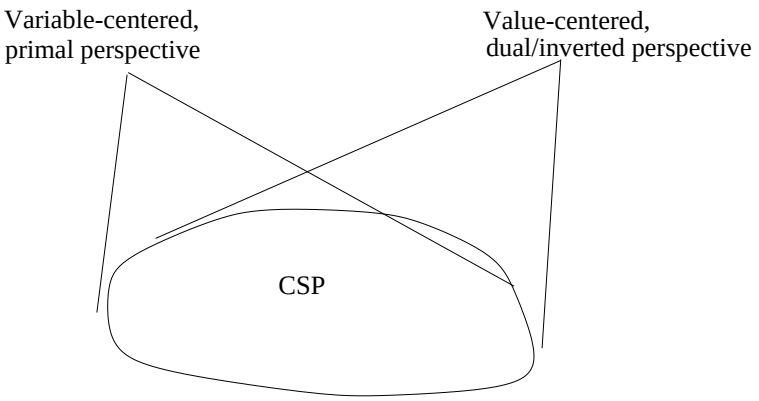
# II- Permutation problems

$\rightarrow$ $\begin{cases} n = a \\ \text{constraint graph is complete} \\ \text{constraints are MUTEX, All-diffs} \end{cases}$

$\rightarrow$ matching, marriage problem

find one value for every variable and exactly one variable for every value

Viewpoints: variable vs. values




$\rightarrow$ Which viewpoint to take?

$\rightarrow$ How to combine computations in viewpoints?

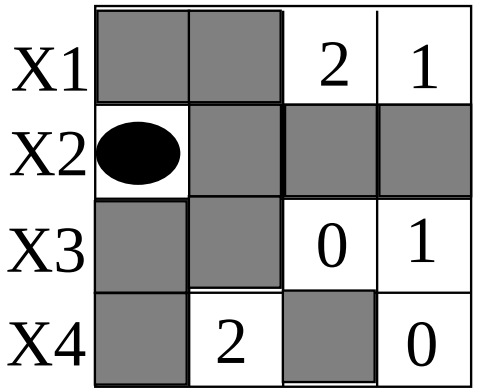
## Permutation problems (cont'd)

At any point in BT,

$$\#\text{ future variables} = \#\text{ future values}$$

$\rightarrow$ choose the most constrained variable in the var-viewpoint, except when the most constrained value in the val-viewpoint is more constrained

## PP: Least domain



# PP: Full evaluation

Introduce: $\begin{cases} \text{LEFT}^{inv}(V_i | X_i = V_i) \\ \text{Promise}^{inv}(V_i = X_i) \end{cases}$

Combine viewpoints: $\text{CPromise}(X_i = V_i)$ min. $\begin{cases} \text{Promise}(X_i = V_i) \\ \text{Promise}^{inv}(V_i = X_i) \end{cases}$

Evaluate vars/vals using: $\begin{cases} \text{CPromise}(X_i) = \sum_k \text{CPromise}(X_i = V_k) \\ \text{CPromise}^{inv}(V_i) = \sum_k \text{CPromise}(X_k = V_i) \end{cases}$

## Partial permutation problems $(n \leq a)$

$\rightarrow$ fake a real PP with bogus variables

$\rightarrow$ extend (3) for PPP

# III- Experiments

- 100 $N$-queen problems: $4 \leq N \leq 103$
- Comparison criteria
  - average number of backtrack
  - number of backtrack-free solutions
  - maximum number of backtracks
  - number of constraint checks

Algorithms:
- LD-1, LD-2, LD-3, LD-1+Dual
- FE-2-4, FE-3-5, FE-3-5+Dual
- FP-2-4, FP-3-5

| Algorithm | Average #backtrack | #backtrack-free solutions | Max. #bactrack |
|---|---|---|---|
| LD+formula 1 | >45000 | 20 | >2500000 |
| LD+formula 2 | >33000 | 15 | >2500000 |
| LD+formula 3 | >1205 | 3 | 92379 |
| LD+formula 1, dual | 21.6 | 26 | 548 |
| FE+formulae 2&4 | 9.5 | 71 | 812 |
| FE+formulae 3&5 | 5.1 | 68 | 266 |
| FP+formulae 2&4 | 5.6 | 81 | 496 |
| FE+formulae 3&5 | 4.2 | 68 | 224 |
| FE+formulae 3&5, dual | 0.38 | 90 | 12 |

Dual perspective exhibits dramatic improvements

CPU time and #CC: plot hard to read, not interpreted:

→ Number of #CC is prohibitive in practice

→ Computations do FC implicitly.

Can be exploited by bookkeeping.