Some characterizations of γ and β-acyclicity of hypergraphs

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Abstract

The notions of γ and β-acyclicity are two classic generalizations of the acyclicity of graphs to hypergraphs. They satisfy the property that, if a hypergraph is γ-acyclic then it is β-acyclic, and the reverse is false. We give some new properties concerning these notions. First we show that we can strictly insert another notion of acyclicity between them, namely the fact of having a join tree with disjoint branches. And if we add a condition on the existence of such a join tree, we obtain a notion equivalent to γ-acyclicity. Then we present two characterizations, consisting in applying successively a small set of rules, deciding γ and β-acyclicity respectively.

1 Introduction

A graph is acyclic if it contains no cycle. On hypergraphs, there exists different definitions of a cycle and thus different non-equivalent notions of acyclicity. For instance, we can cite in increasing order of generality (cf. [Fag83]): γ, β and α-acyclicity. They have been studied in the context of hypergraph theory, database theory and constraint satisfaction problems. For example, γ-acyclic database schemes have interesting structural properties (see for instance [Fag83], [LL89] and [ZC02]) and α-acyclic conjunctive queries on databases (or acyclic queries) form an important tractable subclass of the class of conjunctive queries. Acyclic queries can be recognized in linear time (see [TY84]) and various efficient algorithms have been developed to evaluate them (see for instance [Yan81], [PY99], [GP01] and [BDG07]). In this paper, we focus on γ and β-acyclicity for which we give alternative characterizations. We first introduce the notion of join tree with disjoint branches and show that it provides a new measure of acyclicity which is more general than γ-acyclicity and less general than β-acyclicity. Then, we show that γ-acyclic hypergraphs are precisely the hypergraphs that admit a join tree with disjoint branches with any hyperedge as root. In the second part of the paper, we give rule-based characterizations of γ and β-acyclicity. More precisely, we give two sets of rules $\mathcal{R}_\gamma$ and $\mathcal{R}_\beta$ such that a hypergraph is γ-acyclic (resp. β-acyclic) iff, after applying the rules $\mathcal{R}_\gamma$ (resp. $\mathcal{R}_\beta$) as long as possible, we obtain the empty hypergraph. Such a set of rules characterizing a notion of acyclicity can help to give a better intuition of it. This is also a convenient way to compare these different notions. There already exist such rules characterizing α-acyclicity (cf. [BFMY83]) and γ-acyclicity (cf. [DM82]), but our set of rules for γ-acyclicity is smaller and cannot be obtained from these previous rules.

2 Preliminaries

A *hypergraph* is a couple $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ consisting of a finite set $\mathcal{V}$ and a set $\mathcal{E}$ of non-empty subsets of $\mathcal{V}$. The elements of $\mathcal{V}$ are called *vertices* and those of $\mathcal{E}$ are called *hyperedges*. The *arity* of a hyperedge is its size.

A γ-*cycle* in a hypergraph is a sequence $(E_1, x_1, ..., E_n, x_n)$ $(n \geq 3)$ where the E_is are distinct hyperedges and the x_is distinct vertices, and satisfying the following properties :

- for all $i \in [1, n-1]$, x_i belongs to E_i and E_{i+1} and no other E_j and
- x_n belongs to E_n and E_1 (and possibly to other E_js).

A hypergraph is γ-*acyclic* if it contains no γ-cycle.

A β-*cycle* in a hypergraph is a sequence $(E_1, x_1, ..., E_n, x_n)$ $(n \geq 3)$ where the E_is are distinct hyperedges and the x_is distinct vertices, and satisfying the following property : for all $i \in [1, n]$, x_i belongs to E_i and E_{i+1} and no other E_j (we identify E_{n+1} with E_1). A hypergraph is β-*acyclic* if it contains no β-cycle.

The definitions of γ and β-acyclicity are quite similar. The difference concerns possibly the last vertex of the cycle. It is obvious that γ-acyclicity implies β-acyclicity but these notions are not as close as they can seem. There are various properties that they do not share (see for instance [Fag83], [Leh85], [CZ02] and [Dur08]). However, on graphs (i.e. when the maximum arity is 2), the two notions collapse and coincide with the classic acyclicity.

A *join tree* for a hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ is, if it exists, a rooted tree $T = (\mathcal{E}, \mathcal{J})$ with set of nodes the hyperedges of $\mathcal{H}$ and such that, for every $v \in \mathcal{V}$, the set of nodes of T that contain v is connected in T. A hypergraph is α-*acyclic* if it has a join tree. We know that any β-acyclic hypergraph is also α-acyclic.

3 Join tree with disjoint branches

Definition 3.1. We say that a join tree T of a hypergraph $\mathcal{H}$ has *disjoint branches* if hyperedges of $\mathcal{H}$ belonging to different branches of T are disjoint.

The following two propositions show that having a join tree with disjoint branches for a hypergraph is a notion located between γ and β-acyclicity.

Proposition 3.2. *If a hypergraph is γ-acyclic, it has a join tree with disjoint branches.*

Proof. We prove in fact a stronger result : for every γ-acyclic hypergraph $\mathcal{H}$ and every hyperedge E of $\mathcal{H}$, $\mathcal{H}$ has a join tree with disjoint branches whose root is E. We prove this by induction on the number of hyperedges. This is obviously true when the hypergraph has just one hyperedge. Let $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ be a γ-acyclic hypergraph and assume the induction hypothesis is true for hypergraphs with strictly less hyperedges than $\mathcal{H}$. Let E be any hyperedge of $\mathcal{H}$. We must show that $\mathcal{H}$ has a join tree T with disjoint branches and with E as root.

First, we split the hypergraph $\mathcal{H}$ minus E

$$(\mathcal{V}, \mathcal{E} \setminus E)$$

in connected components $\mathcal{H}_1$, ..., $\mathcal{H}_n$ (we do not consider in these components the vertices that belong only to the hyperedge E). Each of the subhypergraphs $\mathcal{H}_1$, ..., $\mathcal{H}_n$ is γ-acyclic and has strictly less hyperedges than $\mathcal{H}$. Thus, by induction hypothesis, each $\mathcal{H}_i := (\mathcal{V}_i, \mathcal{E}_i)$ has a join tree with disjoint branches and we can choose any hyperedge of $\mathcal{H}_i$ as root. We now specify which hyperedge we will choose and define a join tree T for $\mathcal{H}$ with disjoint branches and with E as root.

For every i, there exists $E_i \in \mathcal{E}_i$ such that $\mathcal{V}_i \cap E \subset E_i$. Indeed, if there was two hyperedges $E_{i,1}$ and $E_{i,2}$ in $\mathcal{E}_i$ such that $E_{i,1} \cap E$ and $E_{i,2} \cap E$ were incomparable for inclusion (i.e. there is a t_1 in $(E_{i,1} \setminus E_{i,2}) \cap E$ and a t_2 in $(E_{i,2} \setminus E_{i,1}) \cap E$) then there would be in $\mathcal{H}$ a γ-cycle beginning with $(E_{i,1}, t_1, E, t_2, E_{i,2}, ...)$ and continuing with any path from $E_{i,1}$ to $E_{i,2}$ of minimal length (possibly just one vertex if $E_{i,1}$ and $E_{i,2}$ intersect). For every i, let T_i be a join tree with disjoint branches for $\mathcal{H}_i$ with E_i as root. We define the join tree T we are looking for as follows. The root of T is E and we connect E to each T_i with an edge $\{E, E_i\}$ (cf. Figure 1). Since the $\mathcal{V}_i$s are pairwise disjoint (by definition of a connected component) and each T_i has disjoint branches, T has disjoint branches. It remains to prove that T is a join tree i.e. that, for every $v \in \mathcal{V}$, the set of hyperedges of $\mathcal{H}$ that contain v is connected in T.

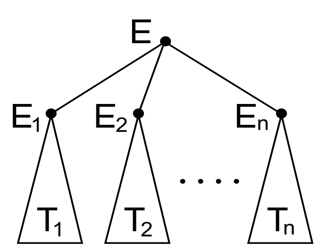

Figure 1: The tree T in the proof of Proposition 3.2.

If $v \in E$ and v belongs to no other hyperedge, this is obvious. If $v \in \mathcal{V} \setminus E$, v belongs to some $\mathcal{V}_i$ and the only hyperedges that contain v are in T_i. Thus, by connectedness of the set of hyperedges that contain v in T_i, we have also the connectedness in T. The remaining case is when v belongs to E and some $\mathcal{V}_i$. Since $\mathcal{V}_i \cap E \subset E_i$, we have $v \in E \cap E_i$. The set of hyperedges S_v^i that contain v in T_i is connected and E_i is in S_v^i. Moreover, the only hyperedge not in T_i that contain v is E and it is connected to E_i in T. Thus, the set $\{E\} \cup S_v^i$ of hyperedges of $\mathcal{H}$ that contain v is connected. □

The preceding proof is constructive. It even provides a polynomial time algorithm that constructs, given a γ-acyclic hypergraph $\mathcal{H}$ and a hyperedge E of $\mathcal{H}$, a join tree of $\mathcal{H}$ with disjoint branches whose root is E. This algorithm would run as follows. For each hyperedge F intersecting E, we check if it is an E_i as in the proof (i.e. $F \cap E$ is maximal for inclusion) and if it is the first candidate we have found (i.e. the preceding hyperedges have not the same intersection with E as $F \cap E$) because we do not want to connect E to T_i several times. For every F satisfying these conditions (F is an E_i), we connect E to the result of the algorithm applied recursively to $\mathcal{H}$ and F (this result is T_i).

Proposition 3.3. *If a hypergraph has a join tree with disjoint branches, it is β-acyclic.*

Proof. Let $\mathcal{H}$ be a hypergraph with a β-cycle $(E_1, x_1, ..., E_n, x_n)$ and let T be a join tree with disjoint branches for $\mathcal{H}$. We will obtain a contradiction. Since E_1 and E_2 intersect, they must belong to the same branch of T. Without loss of generality, we can assume that E_1 is closer to the root of T than E_2. Indeed, if the contrary was true, we could consider instead the β-cycle

$$(E_2, x_1, E_1, x_n, E_n, x_{n-1}, E_{n-1}, ..., E_3, x_2).$$

Now E_3 must be on the same branch as E_2 because they intersect. Since E_1 is above E_2 (if we consider the root as the "top" of the tree), E_1 and E_3 are also on this same branch. Moreover, E_3 must be under E_2 (i.e. E_2 is closer to the root), else we would have a contradiction :

- if E_3 was between E_1 and E_2 then x_1 should belong to E_3 (indeed $x_1 \in E_1 \cap E_2$ and the set of hyperedges containing x_1 is connected in T) and
- if E_3 was above E_1 then x_2 should belong to E_1 (for the same reasons).

With the same arguments, we show that every E_i must belong to this same branch and in the order E_1, E_2, ..., E_n from the root of T. In particular E_n is the farthest and thus, for instance, E_2 is between E_1 and E_n. But, again, x_n belongs to E_1 and E_n and the set of hyperedges containing x_n is connected in T. So x_n should belong to E_2, which is a contradiction. □

Note that the reverse is false in both preceding propositions. Indeed, the hypergraph

$$(\{1, 2, 3\}, \{\{1, 2, 3\}, \{1, 2\}, \{2, 3\}\})$$

contains the γ-cycle $(\{1, 2\}, 1, \{1, 2, 3\}, 3, \{2, 3\}, 2)$ and has a join tree with disjoint branches (cf. Figure 2). And the hypergraph

$$(\{1, 2, 3, 4\}, \{\{1, 2, 3, 4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}\})$$

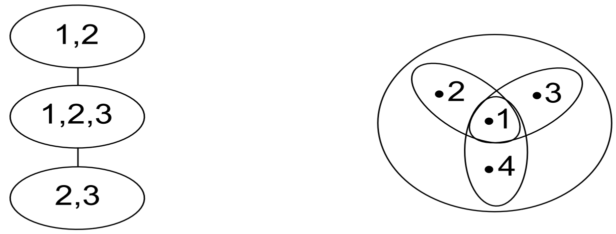

Figure 2: A join tree with disjoint branches for a γ-cyclic hypergraph and a β-acyclic hypergraph that has no join tree with disjoint branches.

is β-acyclic and we can easily check that it has no join tree with disjoint branches.

In the proof of Proposition 3.2, we have seen that we can take any hyperedge as root of the join tree with disjoint branches. Since we will show that the reverse is also true, we obtain the following characterization of γ-acyclicity.

Proposition 3.4. *A hypergraph $\mathcal{H}$ is γ-acyclic if and only if, for every hyperedge E of $\mathcal{H}$, $\mathcal{H}$ has a join tree with disjoint branches whose root is E.*

Proof. As already noticed, the proof of 3.2 yields the only if part. It remains to prove that, if $\mathcal{H}$ has a γ-cycle, there is a hyperedge that cannot be the root of a join tree with disjoint branches for $\mathcal{H}$. Since a γ-cycle is either a β-cycle or a γ-cycle of size 3 that is not a β-cycle (see [Fag83]), we can distinguish between these two cases. Proposition 3.3 says that, if $\mathcal{H}$ has a β-cycle, then no hyperedge can be the root of a join tree with disjoint branches for $\mathcal{H}$. So it remains to study the second case. Let (A, a, B, b, C, c) be a γ-cycle of $\mathcal{H}$ of size 3 that is not a β-cycle. This means that $a \in (A \cap B) \setminus C$, $b \in (B \cap C) \setminus A$ and $c \in A \cap B \cap C$. We show that B cannot be the root of a join tree with disjoint branches for $\mathcal{H}$. If B was the root, A and C should be in the same branch because they are not disjoint ($c \in A \cap C$). On this branch, A cannot be under C (i.e. farther from the root B), because else a would be in C (by connectedness of hyperedges containing a in the tree). Similarly C can neither be under A because this time b would be in A. This is a contradiction. $\square$

It seems hard to find such a condition on join trees that would characterize β-acyclicity. Indeed, if we add a vertex to every hyperedge of a hypergraph, we do not modify its β-acyclicity. The same holds even if we do not add the vertex to every hyperedge but only to some hyperedges chosen cautiously. The situation is quite different for γ-acyclicity since adding a vertex to each hyperedge has much chance to create a γ-cycle.

We finish this section with two corollaries of Proposition 3.2.

Corollary 3.5. *Let $k \geq 1$ be an integer. Every γ-acyclic hypergraph of maximum arity at most k has a join tree of maximum degree at most $k+1$.*

Proof. Let $\mathcal{H}$ be a γ-acyclic hypergraph. By Proposition 3.2, it has a join tree T with disjoint branches. Let E be a vertex of T (i.e. a hyperedge of $\mathcal{H}$) with at most k elements. By the pigeonhole principle and since the children of E in T are disjoint, E has at most k

children that intersect it. Moreover, if a child C of E does not intersect E, we can remove the subtree T_C under C (i.e. the subtree of T induced by C and all its descendants in T) and hook it to any leaf. The tree we obtain is still a join tree of $\mathcal{H}$ with disjoint branches (because the vertices of T_C are disjoint from the other vertices of T). If we proceed this way with every hyperedge E, we get a join tree of $\mathcal{H}$ with disjoint branches of maximum degree at most $k+1$. □

When $k = 2$, this is the special case of acyclic graphs.

Corollary 3.6. *Every acyclic graph has a join tree of maximum degree at most* 3.

Notice that the join tree obtained in the proof of this corollaries still have disjoint branches.

4 Rule-based characterizations

In this section, we present two sets of rules such that applying them successively as long it is possible allows to decide respectively if a hypergraph is γ-acyclic and β-acyclic. These kinds of rules can be convenient to understand better these notions as for the rules for α-acyclicity (remove any vertex belonging to at most one hyperedge and any hyperedge included in another hyperedge, cf. [BFMY83]). They can also be seen as algorithms deciding γ and β-acyclicity.

Let $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ be a hypergraph. We consider the three following rules :

1. If a vertex is isolated (i.e. belongs to only one hyperedge) or belongs to no hyperedge, then we remove it from $\mathcal{H}$ (i.e. from $\mathcal{V}$ and from the hyperedge that contains it if it is isolated).

2. If a hyperedge E satisfies the property : $\forall F \in \mathcal{E}\ (E \subset F \vee E \cap F = \emptyset)$, then we remove it from $\mathcal{E}$ (remark: we only remove E from the list of hyperedges not the vertices of E).

3. If two hyperedges have the same elements, then we remove one of them from $\mathcal{E}$ (remark : when applying the rules, it happens that we do not have a hypergraph any more, because two hyperedges can then have the same elements ; this is why $\mathcal{E}$ is considered as a multi-set).

Proposition 4.1. *A hypergraph $\mathcal{H}$ is γ-acyclic if and only if, after applying the three rules successively until none can be applied, we obtain the empty hypergraph (i.e. with no hyperedge and no vertex).*

Proof. If $\mathcal{H}$ contains a γ-cycle $(E_1, x_1, ..., E_n, x_n)$, then Rule 1 can't remove an x_i, because they all belong to at least two hyperedges (E_i and E_{i+1}) (remark : we assume that the index $n+1$ corresponds to 1). Rule 2 doesn't remove an E_i, because each E_i is connected to at least one other without being included (E_{i+1} for example). And Rule 3 can at most replace an E_i by a hyperedge with the same elements. Consequently, after applying the three rules successively, there will always remain at least one γ-cycle in $\mathcal{H}$, and so we won't obtain the empty hypergraph.

Reciprocally, let $\mathcal{H}$ be a hypergraph such that, if we apply successively 1, 2 and 3, we obtain a non-empty hypergraph $\mathcal{H}'$ to which we cannot apply 1, 2 or 3 anymore. We will show that $\mathcal{H}'$ contains a γ-cycle (and thus that $\mathcal{H}$ contains a γ-cycle). Let A be any hyperedge of $\mathcal{H}'$. Since we cannot apply 2, A intersects a hyperedge B without being included in it. Among the possible Bs, we choose a B that is minimal for inclusion.

If B is not included in A, there exists $a \in A \cap B$ and $b \in B \setminus A$. If B is included in A, since we cannot apply 2 to B, there exists a B' that intersects B and does not contain B. Moreover B' cannot be included in B because B is minimal among hyperedges intersecting A and not containing A. Thus there exists $u \in B \cap B'$, $v \in A \cap (B \setminus B')$ and $w \in B' \setminus B$. If w was in A, then

$$(B, v, A, w, B', u)$$

would be a γ-cycle of $\mathcal{H}'$ and we could already conclude. In every case (even if we must replace B by B' and b by w), there exists $a \in A \cap B$ and $b \in B \setminus A$. Before we continue, we replace B by a $\tilde{B}$ maximal for inclusion, containing B and such that $A \cap \tilde{B} = A \cap B$.

Since b is not isolated (because we cannot apply 1), there exists a hyperedge $C \neq B$ that contains b. We choose C minimal for l'inclusion among the different candidates. We can assume that $B \not\subset C$ for we would else have $A \cap B \subsetneq A \cap C$ (because we have replaced B by $\tilde{B}$), thus there would be a $z \in (A \cap C) \setminus B$ and

$$(A, z, C, b, B, a)$$

would be a γ-cycle. So we can use the same argument as in the preceding paragraph with the hyperedges C and B instead of B and A. We conclude that there exists $b \in B \cap C$ and $c \in C \setminus B$.

We continue to define hyperedges this way until the sequence $A, B, C, ...$ contains the same hyperedge twice. We finally obtain a sequence

$$\mathcal{S} \;\; := \;\; (E_1, x_1, ..., E_n, x_n)$$

such that, for every i, $x_i \in E_i \cap E_{i+1}$ (with $E_{n+1} := E_1$) and, for every $i \in [2, n]$, $x_i \notin E_{i-1}$. We choose $\mathcal{S}$ with minimal size among all possible sequences we could obtain, and we will show that this $\mathcal{S}$ gives rise to a γ-cycle.

First, we have necessarily $n \geq 3$ because $x_2 \notin E_1$. If $n = 3$, then

$$(E_2, x_2, E_3, x_3, E_1, x_1)$$

is a γ-cycle, because $x_2 \in (E_2 \cap E_3) \setminus E_1$, $x_3 \in (E_3 \cap E_1) \setminus E_2$ and $x_1 \in E_1 \cap E_2$. The remaining case is $n \geq 4$. We show that $\mathcal{S}$ is a γ-cycle i.e. that, for every i, x_i belongs to no other E_j than E_i and E_{i+1}.

We notice that, for every $j \in [2, n-1]$, $x_j \notin E_1$ because

$$(E_1, x_1, ..., E_j, x_j)$$

would else contradict the minimality of $\mathcal{S}$. We can now show that, for every $j \in [3, n]$, $x_1 \notin E_j$. Indeed, for every $j \in [3, n-1]$, $x_1 \notin E_j$ because

$$(E_1, x_1, E_j, x_j, ..., E_n, x_n)$$

would also contradict the minimality of $\mathcal{S}$ since $x_j \notin E_1$. And $x_1 \notin E_n$ because else this is

$$(E_2, x_2, ..., E_{n-1}, x_{n-1}, E_n, x_1)$$

that would contradict the minimality of $\mathcal{S}$ since we have just seen that $x_1 \notin E_{n-1}$.

We now consider the sequence

$$(E_2, x_2, E_3, x_3, ..., E_n, x_n, E_1, x_1).$$

As $x_1 \notin E_n$, the preceding argument shows that, for every $j \in [4, n] \cup \{1\}$, $x_2 \notin E_j$. Then, considering the sequence

$$(E_3, x_3, E_4, x_4, ..., E_n, x_n, E_1, x_1, E_2, x_2),$$

we obtain that, for every j different from 3 and 4, $x_3 \notin E_j$. And so on, we have that, for every i, x_i belongs to no other E_j than E_i and E_{i+1}, which is what we needed. □

Notice that the preceding proof contains the proof of the following fact.

Fact 4.2. *If a hypergraph $\mathcal{H}$ has a sequence $(E_1, x_1, ..., E_n, x_n)$ such that,*

$$\text{for every } i,\ x_i \in E_i \cap E_{i+1} \text{ (with } E_{n+1} := E_1) \quad \text{and} \quad \text{for every } i \in [2, n],\ x_i \notin E_{i-1},$$

then $\mathcal{H}$ has a γ-cycle.

Now we consider the two following rules :

1. If a vertex is a nest point (i.e. the set of hyperedges containing it is a chain for the inclusion relation), then we remove it from $\mathcal{H}$ (i.e. from $\mathcal{V}$ and from the hyperedges that contain it).

2. If a hyperedge is empty, then we remove it from $\mathcal{E}$.

In [BK80], Brouwer and Kolen show that a hypergraph is β-acyclic if and only if every induced subhypergraph has a nest point. So, applying the rules successively on a β-acyclic hypergraph will yield the empty hypergraph. And conversely, we see easily that, if a hypergraph has a β-cycle, we will not be able to apply the rules on the elements of the β-cycle, and thus we will not obtain the empty hypergraph. Consequently, we have the following proposition.

Proposition 4.3. *A hypergraph $\mathcal{H}$ is β-acyclic if and only if, after applying the two rules successively until none can be applied, we obtain the empty hypergraph.*

It is interesting to notice that, if we want to test β-acyclicity, we thus not have to search for a nest point in every induced subhypergraph, but only in $|\mathcal{V}|$ induced subhypergraph.

5 Conclusion

We have seen the position of join trees with disjoint branches in the hierarchy of hypergraph acyclicity. Since we know that having a join tree (with no additional condition) is equivalent to being α-acyclic, we have the following recap :

$$jtdb \text{ for any root} \;=\; \gamma\text{-acyclicity} \;\subsetneq\; jtdb \;\subsetneq\; \beta\text{-acyclicity} \;\subsetneq\; \alpha\text{-acyclicity} \;=\; jt,$$

where jt means "join tree" and db "disjoint branches".

Inspired from the notion of join tree with disjoint branches, it could be interesting to investigate the condition of disjoint branches in classical decomposition methods, such as hypertree decompositions (cf. [GLS02]). These decompositions are very useful in the areas of databases and CSPs since recognize queries of bounded hypertree width, construct a hypertree decomposition for them and evaluate them are all feasible in polynomial time.

The rule-based characterizations of γ and β-acyclicity give the idea of studying further "local conditions", for instance conditions concerning the set of hyperedges containing a vertex (as in the case of β-acyclicity where they form a chain for inclusion).

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References

[BDG07] G. Bagan, A. Durand, and E. Grandjean. On acyclic conjunctive queries and constant delay enumeration. In *Proceedings of the 16th EACSL Annual Conference on Computer Science and Logic (CSL 2007)*, LNCS 4646, pages 208–222, 2007.

[BFMY83] C. Beeri, R. Fagin, D. Maier, and M. Yannakakis. On the desirability of acyclic database schemes. *J. of the ACM*, 30(3):479–513, 1983.

[BK80] A. E. Brouwer and A. W. J. Kolen. A super-balanced hypergraph has a nest point. Technical report, Math. centr. report ZW146, Amsterdam, 1980.

[CZ02] Y. Chunlong and H. Zhongxiao. The existence condition of gamma-acyclic database schemes with mvds constraints. *J. of Computer Science and Technology*, 17:517–521, 2002.

[DM82] A. D'Atri and M. Moscarini. Acyclic hypergraphs: their recognition and top-down versus bottom-up generation. Technical Report R. 29, Consiglio Nazionale Delle Richerche, Istituto di Analisi dei Sistemi ed Informatica, 1982.

[Dur08] D. Duris. Hypergraph acyclicity and extension preservation theorems. In *Proceedings of the 23rd Annual IEEE Symposium on Logic in Computer Science (LICS 2008)*, pages 418–427, 2008.

[Fag83] R. Fagin. Degrees of acyclicity for hypergraphs and relational database schemes. *J. of the ACM*, 30(3):514–550, 1983.

[GLS02] G. Gottlob, N. Leone, and F. Scarcello. Hypertree decompositions and tractable queries. *J. of Computer and System Sciences*, 64(3):579–627, 2002.

[GP01] G. Gottlob and R. Pichler. Hypergraphs in model checking: acyclicity and hypertree-width versus clique-width. In *Proceedings of the 28th International Colloquium on Automata, Languages and Programming (ICALP 2001)*, number 2076 in LNCS, pages 708–719, 2001.

[Leh85] J. Lehel. A characterization of totally balanced hypergraphs. *Discrete mathematics*, 57:59–65, 1985.

[LL89] M. Levene and G. Loizou. gamma-acyclic database schemes and nested relations. *Lecture Notes in Computer Science*, 361:313–323, 1989.

[PY99] C. H. Papadimitriou and M. Yannakakis. On the complexity of database queries. *J. of computer and system sciences*, 58:407–427, 1999.

[TY84] R. E. Tarjan and M. Yannakakis. Simple linear-time algorithms to test chordality of graphs, test acyclicity of hypergraphs, and selectively reduce acyclic hypergraphs. *SIAM Journal on computing*, 13:566–579, 1984.

[Yan81] M. Yannakakis. Algorithms for acyclic database schemes. In *Proceedings of the 7th International Conference on Very Large Data Bases (VLDB 1981)*, pages 82–94, 1981.

[ZC02] H. Zhongxiao and Y. Chunlong. The existence condition of gamma-acyclic database schemes with mvds constraints. *Journal of Computer Science and Technology*, 17:517–521, 2002.