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Consistency of triangulated temporal qualitative constraint networks

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***Abstract*—In this paper, we introduce for the qualitative constraint networks (QCNs) a new consistency: the partial $\diamond$-consistency. The partial $\diamond$-consistency, similarly to the partial path-consistency, considers triangles of a graph and corresponds to the $\diamond$-consistency restricted to these triangles. We show that for the pre-convex QCNs of the Interval Algebra (IA), the partial $\diamond$-consistency with respect to a triangulation of the graph of constraints is sufficient to decide the consistency problem. From this result, we propose an algorithm allowing to solve QCNs of IA. The experiments that we have conducted show the interest of this algorithm to solve the consistency problem of the QCNs of IA.**

I. Introduction

A qualitative constraint calculus introduces particular elements to represent the temporal entities of the system and a finite set of base relations over these entities. Each base relation corresponds to a particular configuration between the entities. Typically, qualitative constraint networks (QCN) are used to express information on a global temporal configuration. Each constraint of a QCN represents a set of acceptable qualitative configurations between some temporal entities and is defined by a set of base relations.

Given a QCN, the main problem is to determine whether the QCN is consistent. In general case, this problem is NP-complete. Despite it, this problem can be decided in polynomial time for particular classes of relations as the class of pre-convex relations of IA [7, 9]. An algorithm to solve effectively the consistency problem of a QCN has been proposed by Nebel [8]. It consists in a backtrack search using the method of the closure under weak composition as filtering method for removing some inconsistent base relations. Moreover, at each step of search, instead of splitting a constraint into base relations, a constraint is split into relations belonging to a tractable class. It allows to reduce the branching factor of the search tree.

QCNs are complete in the sense where between each pair of variables is defined a constraint by a relation of the calculus. When all possible configurations are allowed between two variables, the universal relation denoted by Ψ is used. The relation Ψ contains all base relations of the calculus. During search, filtering can remove some base relations from a constraint initially defined by the relation Ψ. In [5], Condotta *et al.* show that we do not need to select such a constraint for splitting during a search based on the Nebel's algorithm.

Relation	Symbol	Converse	Illustration
precedes	p	pi	
meets	m	mi	
overlaps	o	oi	
starts	s	si	
during	d	di	
finishes	f	fi	
equals	eq	eq	

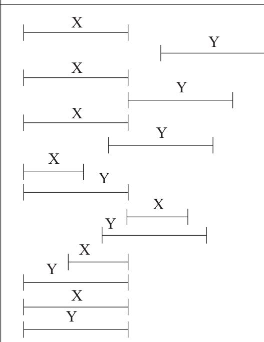

Figure 1. The base relations of IA (B_{IA}).

In this paper, we show that the filtering with the method of closure under weak composition can be just realized on a subset of triangles of variables. For this, we follow the line of reasoning in [3] in which the partial path-consistency (PPC) is proposed. They show that PPC solves convex CSP for which constraint graphs are triangulated. In the paper, we show that we have a similar result for the pre-convex relations of IA.

II. Preliminaries

A qualitative calculus considers a finite set of (binary) relations B, called base relations, over a domain D representing the temporal entities. In this paper, we are mainly concerned by the particular qualitative calculus called the Interval Algebra (IA), also called as Allen's calculus [1]. IA considers the intervals of the line to represent the temporal entities. The domain of IA is defined by $\mathsf{D}_{\mathsf{IA}} = \{(x^-, x^+) \in \mathbb{Q} \times \mathbb{Q} \mid x^- < x^+\}$. The base relations of IA correspond to the set $\mathsf{B}_{\mathsf{IA}} = \{eq, p, pi, m, mi, o, oi, s, si, d, di, f, fi\}$. The elements of B_{IA} represent all possible orderings between the four bounds of two intervals on the line. In Figure 1 are illustrated these base relations.

A complex relation, also called relation, is an union of base relations. It is customary to represent a relation by the set of the base relations which compose it. Hence, the set 2^{B} which will represent the set of relations of a qualitative calculus based on the set of base relations B. Given two elements $x, y \in \mathsf{D}$ and a base relation $b \in \mathsf{B}$ (resp. a relation $r \in 2^{\mathsf{B}}$,

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$x\ b\ y$ (resp. $x\ r\ y$) will denote the fact that x and y satisfies b (resp. a base relation of r).

The usual set-theoretic operations union ($\cup$), intersection ($\cap$) are defined on 2^{B}. The set 2^{B} is also equipped with the converse operation ($^{-1}$) and the weak composition operation ($\diamond$). For a relation $r \in 2^{\mathsf{B}}$, the converse relation of a relation r is the union of the converse relations of its base relations: $r^{-1} = \{b^{-1} | b \in r\}$. The weak composition operation of two relations $r, s \in 2^{B}$ is defined by: $a \diamond b = \{c \in \mathsf{B} : \exists x, y, z \in \mathsf{D}$ with $x\ a\ z, z\ b\ y$ and $x\ c\ y\}$, with $a, b \in \mathsf{B}$; $r \diamond s = \bigcup_{a \in r, b \in s} \{a \diamond b\}$, with $r, s \in 2^{\mathsf{B}}$. Among the relations of 2^{B}, Ψ denotes the particular relation that contains all the base relations of B. A class of relations $\mathcal{C}$ is a subset of 2^{B} which contains the relation Ψ, all of the singleton relations of 2^{B}, and which is closed under converse, intersection and weak composition. Given $r \in 2^{\mathsf{B}}$ and a class $\mathcal{C}$, the smallest relation of $\mathcal{C}$ which contains r is denoted by $\mathcal{C}(r)$ and is called the closure of r in $\mathcal{C}$. Now, we introduce some particular classes of IA: the set of the convex relations and the set of the pre-convex relations (also called ORD-Horn relations in [9]). Ligozat [6] arranges the base relations of IA in a partial order which defines a lattice $(\mathsf{B}_{\mathsf{IA}}, \leq)$. To define pre-convex relations, Ligozat attributes a dimension to each base relation of IA. Intuitively, this dimension corresponds to 2 minus the number of equalities of bounds of two intervals satisfying the base relation considered. Hence, the dimension of the base relations p, pi, o, oi, d, di is 2, this one of the base relations m, mi, s, si, f, fi is 1, and the dimension of eq is 0. The dimension of a relation $r \in 2^{\mathsf{B}_{\mathsf{IA}}}$ is the maximal dimension of its base relations. A pre-convex relation is a relation of IA such that its closure *w.r.t.* the class of the convex relations does not contain a new base relation with a dimension equals or greater than its dimension.

A Qualitative Constraint Network (QCN) is a pair composed of a set of variables and a set of constraints defined in the following way:

Definition 1: A QCN is a pair (V, C) where:

- $V = \{v_0, \ldots, v_{n-1}\}$ is a set of n variables representing temporal entities,
- C is a map associating a relation $C(v_i, v_j) \in 2^{\mathsf{B}}$ with each pair of variables $(v_i, v_j) \in V \times V$. C is such that $C(v_i, v_i) \subseteq \{\mathsf{Id}\}$ and $C(v_i, v_j) = C(v_j, v_i)^{-1}$ for all $v_i, v_j \in V$.

In the sequel, we will also denote $C(v_i, v_j)$ by $\mathcal{N}[v_i, v_j]$. Given a QCN $\mathcal{N} = (V, C)$, a partial instantiation of $\mathcal{N}$ on $V' \subseteq V$ is a map s from V' to D. A partial solution of $\mathcal{N}$ on $V' \subseteq V$ is a partial instantiation on V' such that $(s(v_i), s(v_j))$ satisfies $C(v_i, v_j)$ for all $v_i, v_j \in V'$. A solution of $\mathcal{N}$ is a partial solution of $\mathcal{N}$ on V. $\mathcal{N}$ is consistent if, and only if, there exists a solution of $\mathcal{N}$. $\mathcal{N}$ is trivially inconsistent when there exist two variables $v, v' \in V$ such that $\mathcal{N}[v, v'] = \emptyset$. $\mathcal{N}$ is globally consistent if, and only if, each partial solution of $\mathcal{N}$ can be extended to a solution of $\mathcal{N}$. The projection of the QCN $\mathcal{N}$ to V' with $V' \subseteq V$, denoted by $\mathcal{N}_{V'}$, is the QCN (V', C_{proj}) with C_{proj} the restriction of C to the set V'. A subQCN $\mathcal{N}'$ of $\mathcal{N}$ is a QCN (V, C') such that $C'(v_i, v_j) \subseteq C(v_i, v_j)$, for all $v_i, v_j \in V$. Let $\mathcal{N}^1$ and $\mathcal{N}^2$ be two QCNs defined on the same set of variables V. We denote by $\mathcal{N}^1 \cup \mathcal{N}^2$ the unique QCN $\mathcal{N}^3$ defined on V by $\mathcal{N}^3[v, v'] = \mathcal{N}^1[v, v'] \cup \mathcal{N}^2[v, v']$ for all $v, v' \in V$. Given a class of relations $\mathcal{C}$, the closure of $\mathcal{N}$ *w.r.t.* $\mathcal{C}$, denoted by $\mathcal{C}(\mathcal{N})$, is the QCN $\mathcal{N}'$ defined on V by $\mathcal{N}'[v, v'] = \mathcal{C}(\mathcal{N}[v, v'])$ for all $v, v' \in V$. A QCN $\mathcal{N}$ is $\diamond$-consistent (we will say also closed by weak composition) if, and only if, $C(v_i, v_j) \subseteq C(v_i, v_k) \diamond C(v_k, v_j)$ for all $v_i, v_j, v_k \in V$. The weak composition closure of the QCN $\mathcal{N}$, denoted by $\diamond(\mathcal{N})$ is the largest (for $\subseteq$) $\diamond$-consistent subQCN of $\mathcal{N}$.

For some classes of relations, such as the class of the pre-convex relations of IA, the consistency problem of a QCN can be decided by enforcing $\diamond$-consistency. Hence, a $\diamond$-consistent pre-convex QCN with no empty constraint is a consistent QCN [9, 7]. The class of convex relations admits a stronger property: each $\diamond$-consistent convex QCN non trivially inconsistent is globally consistent [2]. The $\diamond$-consistent pre-convex QCNs non trivially inconsistent admits a property close of the global consistency. Indeed, in [7] Ligozat considers particular instantiations that we will call maximal instantiations and which are defined as follows: a partial solution of maximal dimension is a solution satisfying for every pair of variables a base relation of maximal dimension with regard to the dimensions of the base relations belonging to the constraint. Ligozat proved that a maximal partial solution can always be extended to a maximal solution. This property has been used to prove some propositions introduced in this paper.

In a natural way we define the constraint graph of a QCN $\mathcal{N} = (V, C)$, by the undirected graph $\mathsf{G}(\mathcal{N}) = (V, E)$ with $(v, v') \in E$ if, and only if, $\mathcal{N}[v, v'] \neq \Psi$ and $v \neq v'$.

Given two graphs $G = (V, E)$ and $G' = (V', E')$, G is a subgraph of G', denoted by $G \subseteq G'$, if, and only if, $V \subseteq V'$ and $E \subseteq E'$. A graph $G = (V, E)$ is a triangulated graph if, and only if, each of its cycles of length strictly greater than 3 has a chord. For a graph $G = (V, E)$, a perfect elimination ordering is an ordering of the vertices V such that, for each vertex $v \in V$, v and the neighbors of v that occur after v in the order form a clique of G. More formally, an ordering of G will be defined by a one-to-one map α which associates to each $i \in \{0, \ldots, |V|-1\}$ a vertex α_i belonging to V. By denoting by, $\overline{\alpha_i}$ the set of vertices $\{\alpha_i\} \cup \{\alpha_j \in V : (\alpha_i, \alpha_j) \in E$ and $j \in \{i+1, \ldots, |V|-1\}\}$, α is a perfect elimination ordering if, and only if, $\overline{\alpha_i}$ is a clique of G for each $i \in \{0, \ldots, |V|-1\}$. In the sequel, a perfect elimination ordering α of $G = (V, E)$ will be sometimes denoted by the sequence $[\alpha_0, \ldots, \alpha_{|V|-1}]$. A graph G is a triangulated graph, if, and only if, G admits a perfect elimination ordering.

III. Partial $\diamond$-consistency and consistency

In the framework of discrete CSPs, partial path consistency (PPC) corresponds to the path consistency restricted to triangles belonging to a graph. In a similar way, we define the partial $\diamond$-consistency of a QCN w.r.t. a graph as the $\diamond$-consistency restricted to triangles of this graph:

Definition 2: Let $\mathcal{N} = (V, C)$ be a QCN and a graph $G = (V, E)$. $\mathcal{N}$ is $\diamond$-consistent with respect to G, denoted by $\diamond_G$-consistent, if, and only if, for every $v_i, v_j, v_k \in V$ such $\{(v_i, v_j), (v_i, v_k), (v_k, v_j)\} \subseteq E$, we have $C_{ij} \subseteq C_{ik} \diamond C_{kj}$.

Proposition 1: Given a QCN $\mathcal{N} = (V, C)$ and a graph $G = (V, E)$, we have: (1) There exists a QCN, denoted by $\diamond_G(\mathcal{N})$ corresponding to the greatest (w.r.t. $\subseteq$) $\diamond_G$-consistent subQCN of $\mathcal{N}$; (2) $\diamond_G(\mathcal{N})$ is an equivalent subQCN of $\mathcal{N}$; (3) $\diamond_G(\diamond_G(\mathcal{N})) = \diamond_G(\mathcal{N})$. (4) Given a $QCN\ \mathcal{N}'$, if $\mathcal{N}' \subseteq \mathcal{N}$ then $\diamond_G(\mathcal{N}') \subseteq \diamond_G(\mathcal{N})$.

We have following properties concerning the $\diamond_G$-consistency for a graph G:

Proposition 2: Let $\mathcal{N} = (V, C)$ a QCN, $G = (V, E)$ a graph and a class of relations $\mathcal{C}$. We have, if $\mathcal{N}$ is $\diamond_G$-consistent then $\mathcal{C}(\mathcal{N})$ is $\diamond_G$-consistent.

Proposition 3: Let $\mathcal{N} = (V, C)$ be a QCN and a triangulated graph $G = (V, E)$ such that $\mathcal{N}$ is a $\diamond_G$-consistent QCN and $\mathsf{G}(\mathcal{N}) \subseteq G$. Given α a perfect elimination ordering of G, for each $i \in \{0, \dots, |V| - 1\}$ we have $\mathcal{N}_{\overline{\alpha_i}}$ which is a $\diamond$-consistent QCN.

Proposition 4: Let $\mathcal{N} = (V, C)$ be a QCN and a triangulated graph $G = (V, E)$. Given a perfect elimination ordering α of G, for each $i \in \{0, \dots, n-1\}$, with $n = |V|$:

(1) we have $\bigcup_{j \in \{i, \dots, n-1\}} \overline{\alpha_j} = \{\alpha_j : j \in \{i, \dots, n-1\}\}$;
(2) if $\mathsf{G}(\mathcal{N}) \subseteq G$ then for each $j \in \{i+1, \dots, |V| - 1\}$ and for each $v \in \overline{\alpha_j} \setminus \overline{\alpha_i}$, we have $C(\alpha_i, v) = \Psi$.

Now, we give an important result concerning the pre-convex QCNs of IA:

Proposition 5: Let $\mathcal{N} = (V, C)$ be a non trivially inconsistent pre-convex QCN of IA and $G = (V, E)$ a graph with $\mathsf{G}(\mathcal{N}) \subseteq G$. If G is a triangulated graph and $\mathcal{N}$ a $\diamond_G$-consistent QCN then $\mathcal{N}$ is a consistent QCN.

IV. Algorithms

In this section, we give an algorithm to decide the consistency problem of a QCN $\mathcal{N}$ of IA by using the class of pre-convex relations and the $\diamond_G$-consistency with G a triangulated graph with $\mathsf{G}(\mathcal{N}) \subseteq G$. First, consider the function PWC (Partial Weak Consistency) which takes as parameters a QCN $\mathcal{N} = (V, C)$, a graph $G = (V, E)$ and possibly an edge $e \in E$. PWC is very close to the algorithm PPC given in [3]. Its objective is to compute the closure of $\mathcal{N}$ for the $\diamond_G$-consistency. In the case where the edge e is given, $\mathcal{N}$ is supposed to be $\diamond_{G'}$-consistent for $G' = (V, E \setminus \{e\})$. By denoting δ the maximal degree of a variable of G, we have for each (v_i, v_j) selected at line 5, at most δ variables v_k of V such that v_i, v_j, v_k forms a triangle. It results that the time complexity of PWC is $O(\delta.|\mathsf{B}|.|E|)$.

Function `PWC(`$\mathcal{N}$,G,e`)`

in : $\mathcal{N} = (V, C)$ a QCN, $G = (V, E)$ a graph, e an edge of G (possibly null).
output : $\diamond_G(\mathcal{N})$

```
1  begin
2      if e ≠ null then Q ← {e};
3      else Q ← E;
4      while Q ≠ ∅ do
5          (v_i, v_j) ← Dequeue(Q);
6          foreach v_k ∈ Q such that
           (v_i, v_j), (v_i, v_k), (v_k, v_j) ∈ E do
7              rel_kj ← N[v_k, v_j] ∩ (N[v_k, v_i] ◇ N[v_i, v_j]);
8              if N[v_k, v_j] ⊈ rel_kj then
9                  Q ← Q ∪ {(v_k, v_j)} ;
10                 N[v_k, v_j] ← rel_kj ; N[v_j, v_k] ← rel_kj^-1 ;
11             rel_ik ← N[v_i, v_k] ∩ (N[v_i, v_j] ◇ N[v_j, v_k]);
12             if N[v_i, v_k] ⊈ rel_ik then
13                 Q ← Q ∪ {(v_i, v_k)} ;
14                 N[v_i, v_k] ← rel_ik ; N[v_k, v_i] ← rel_ik^-1 ;
15     return N
16 end
```

Function `Consistency(`$\mathcal{N}$,$\mathcal{C}$`)`

in : A QCN $\mathcal{N} = (V, C)$, $\mathcal{C}$ a class of relations .
output : $true$ or $false$.

```
1  begin
2      G = (V, E) ← Triangulation(G(N));
3      N_Init ← N; N ← PWC(N, G, null) ;
4      if N = ⊥ then return false ;
5      return ConsistencyAux(N,G) ;
6  end
```

Function `ConsistencyAux(`$\mathcal{N}$,G`)`

in : $\mathcal{N} = (V, C)$ a QCN, $G = (V, E)$ a graph.
output : $true$ or $false$.

```
1  begin
2      Select (v, v') ∈ E such that C(N[v, v'])) ⊈ N_Init[v, v'];
3      if a such pair does not exist then return true ;
4      Split N[v, v'] into sub-relations r_1, ..., r_k ∈ C ;
5      N' ← N ;
6      foreach i ∈ 1, ..., k do
7          N[v, v'] ← r_i; N[v', v] ← r_i^-1;
8          N ← PWC(N, G, (v, v')) ;
9          if N = ⊥ then return false ;
10         if ConsistencyAux(N, G) then return true ;
11         N ← N' ;
12     return false
13 end
```

Consider now the main function Consistency which takes as parameters a QCN $\mathcal{N}$ and a class of relations $\mathcal{C}$. At line 2, Consistency computes a triangulated graph G such that $\mathsf{G}(\mathcal{N}) \subseteq G$. Then, the QCN $\mathcal{N}_{Init}$ is defined by the QCN $\mathcal{N}$ given as parameter. Some base relations of $\mathcal{N}$ are removed

by computing $\diamond_G(\mathcal{N})$ (line 3) with PWC. After this operation $\mathcal{N}$ is a $\diamond_G$-consistent subQCN of $\mathcal{N}_{Init}$ equivalent to $\mathcal{N}$. If $\mathcal{N}$ contains a constraint defined by the empty relation, the inconsistency of the initial QCN is detected (line 4). In the contrary case, the recursive function ConsistencyAux is called with the parameters $\mathcal{N}$ and G.
The function ConsistencyAux takes as parameters a QCN $\mathcal{N}$ and a triangulated graph G such that $G \subseteq \mathsf{G}(\mathcal{N})$, $\mathcal{N} \subseteq \mathcal{N}_{Init}$ and $\mathcal{N}$ must be $\diamond_G$-consistent. The ConsistencyAux searches a $\diamond_G$-consistent subQCN of $\mathcal{N}$ that its closure with respect to the class of relations $\mathcal{C}$ is a subQCN of $\mathcal{N}_{Init}$.

Proposition 6: Given a QCN $\mathcal{N} = (V, C)$ and a class of relations $\mathcal{C}$. The function Consistency stops, and for a triangulated graph $G = (V, E)$ with $\mathsf{G}(\mathcal{N}) \subseteq G$ returns:
-**true** in the case where there exists a non trivially inconsistent subQCN $\mathcal{N}'$ of $\mathcal{N}$ such that $\mathcal{N}'$ is $\diamond_G$-consistent and $\mathcal{N}'[v, v'] \in \mathcal{C}$ for each $(v, v') \in E$.
-**false** in the case where there does not exist a non trivially inconsistent subQCN $\mathcal{N}'$ of $\mathcal{N}$ such that $\mathcal{N}'$ is $\diamond_G$-consistent and $\mathcal{N}'[v, v'] = \{a\}$ with $a \in \mathcal{N}[v, v']$ for each $(v, v') \in E$.
From this we can assert the following theorem:

Theorem 1: For $\mathcal{C}$ the pre-convex class of IA, the function Consistency decides the consistency problem of the QCNs of the IA.

V. Preliminary experimental results

For experimentation we have focused on QCNs of IA randomly generated following Model A [8]. For this model, the generation of QCNs depends on the number of variables (n), the density of non trivial relations (d) and the average number of base relations in each constraint (s). The experimental results realized concern QCN instances from $A(100, d, 6.5)$ for d varying from 4 to 22 with a step of 0.25. For each series, 100 QCNs is generated and a maximal solving time of 4 hours is given. The hardest instances are located in an interval where the density ranges from 8 to 11. This is where a phase transition occurs between under-constrained instances and over-constrained instances. Our program called Sparrow written in language C implements the function Consistency studied in the previous section. Heuristics concerning selection of constraints during search is very close than those used in [8]. The triangulation of graphs is realized by an external program using the Java library LibTW (http://www.treewidth.com/). The technique used to triangulate a graph consists in adding extra edges produced by eliminating vertices one by one. In our experiments we used the *GreedyFillIn* (GIF) one [4] which aims to minimize the number of added edges to the graph.

The main objective of our experimentation is to compare the time efficiency of the algorithm Consistency. We compare our program with GIF triangulation and without triangulation of graphs. We can notice that the use of GIF triangulation allows to restrict considerably the number of pertinent edges and triangles. For example, for a density of non trivial relations of 10, we obtain less than 10 % of useful triangles and less than 40 % of constraints which must be considered during the search. Concerning the solving time, we can observe that these gains lead in better one performance of Sparrow when it uses GIF triangulation. Remark that for the series $d = 10$, just GIF triangulation allows to solve all the instances.

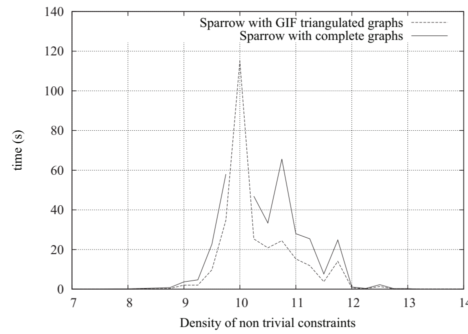

Figure 2. Solving time by Sparrow.

Conclusion

In this paper, we introduce the partial $\diamond$-consistency for QCNs. We show that for the pre-convex QCNs of IA, the partial $\diamond$-consistency with respect to a triangulation of the graph of constraints is sufficient to decide the consistency problem. From this result, we give an algorithm to solve QCNs of IA. A future work consists in using of other methods of triangulations and compare the behavior of our algorithm for this different methods. Also, a possible perspective is to consider the minimal problem of QCNs and define an algorithm to solve this problem by using the partial $\diamond$-consistency.

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