# Average Delay Analysis of Opportunistic Single Copy Delivery in Manhattan Area using Biased Random Walk

Jung Hyun Jun, Weihuang Fu, and Dharma P. Agrawal
School of Computing Sciences and Informatics
University of Cincinnati, Cincinnati, OH 45221-0030
Email: junjn@mail.uc.edu, fuwg@mail.uc.edu, dpa@cs.uc.edu

***Abstract*—In Mobile Opportunistic Networks, the cost and effectiveness of any opportunistic forwarding is measured by the expected delay of a message. Hence, its critical goal is to have low delay for a message. This paper studies the average delay of a message in a Mobile Opportunistic Network on Manhattan area. We first model the mobility of a message as a biased random walk in tilted grid and analyze the delay of a message based on the hitting time of a bias random walk. We have derived an exact expression of expected delay for a walk starting from any point in tilted grid for both biased and unbiased random walks and provide a closed form approximation of average delay of a message for the case of unbiased random walk. The key result is that the average delay of a message in Mobile Opportunistic Networks is very sensitive to the biased level of a random walk at each stage of the walk (depends on the distance from destination at its current stage). Then, this key result explains why most of the smart message forwarding algorithm in Mobile Opportunistic Network works reasonably well.**

## I. Introduction

In Mobile Opportunistic Networks (MONs), a message is carried by randomly moving mobile nodes and any opportunistic connectivity like encountering another mobile node is considered for forwarding of a message until the message reaches the sink. However, the connectivities are intermittent due to the mobility of mobile nodes being independent and random. This lack of connectivities causes potentially long delays for a message in MONs.

The notion of MONs, [1], finds its origin from Delay Tolerant Networks (DTNs), [2], where messages are relayed physically from source to destination by mobile units [3]. Such DTN characteristics appear commonly in a Wireless Sensor Network (WSN), to reduce the multi-hop transmissions from source to sink, since maximizing the life time of network by saving the energy is the most critical issue. Besides the energy efficiency, it gained increasing attention from wireless network researchers when Grossglauser and Tse [4] showed that it is possible to increase the capacity bound $\Theta\left(1/\sqrt{m \log m}\right)$ obtained by Gupta and Kuma [5] to $\Theta\left(1\right)$ with only a single relay node (RN) to sink, where $m$ is the number of identically randomly located wireless nodes. The Gupta and Kuma's capacity bound implies, as the number of node (sensors or Ad-hoc units) increases to infinity, the capacity will approach to zero. That is why Grossglauser and Tse's result still give us some hope to achieve a reasonable throughput in a network with high node density.

Most DTNs have a very small number of RNs as compared to the number of static sensors and they are actual part of the WSN. Their mobilities depend on the network topology or are controlled by the network itself [6]. In MONs, the network often does not have any control over the mobility of mobile units, so their mobilities are random and independent of any underlaying network topology. Additionally the routings of mobile RNs are more generic than DTNs. Therefore, there is no cost of maintaining and controlling their mobilities. It simply considers any opportunistic connection for message forwarding. Despite the potential benefits of MONs, the long message delay due to intermittent connectivity of RNs can be a significant drawback for some applications. For example, surveillance applications need to have some assurance for the delay of message so that the data containing critical (or dangerous) information can quickly reach the sink and get processed. Another example is an industrial or farming related applications which may not require immediate response, but still needs to have some reasonable guarantees in their average delays, so that it is not too late to respond to any potential damages due to early frost or pest attacks.

In a MON, the message delay is significantly influenced by the number of RNs in the network and underlying message forwarding algorithm. We have listed a few routing algorithms in the related works section II. In this paper, we analyze the average delay of message in MONs on Manhattan area. However, instead of analyzing the message delay for different forwarding algorithms, we directly look at the combined effect of the number of RNs and the goodness of forwarding algorithms, to the delay of messages by introducing a single parameter called bias level. The main contributions of this paper are,

- Provide alternative method of estimating the hitting time of a biased (or unbiased) random walk in a tilted square grid, by mapping it to the biased random walk in 1-dimensional Markov chain.
- The exact expression for the expected delay of single copy forwarding in the MON on a tilted grid topology is derived. This expression could be useful for message routing optimization in MONs, such that the minimum

978-1-4244-9920-5/11/$26.00 ©2011 IEEE 

average message delay is satisfied.
- Deriving the bound of expected delay of a message, with respect to the bias level in Theorem 4.1. It reveals that the upper bound of the expected delay of a message is sensitive to bias level and it reduces from quadratic delay to linear with small increases in the bias level. This provides a concrete reason why all the simple single copy forwarding schemes help reduce the delay of messages significantly, compared to random walking (without any smart forwarding). This result is similar to Beraldi [7] who analyzed the average delay of biased random walks in Uniform Wireless Networks.
- The bias level, under our definition, influences the probability that the selected RN is actually moving closer to the sink than the current RN. We show in the Corollary 4.2 that if the bias level of a moving message can be maintained greater than some threshold for the entire trip to the sink, then the delay of the message will stay in linear order of its rectilinear distance from its origin to the sink.

## II. Related Works

Since the delay of message forwarding is unreliable, several works have attempted to reduce the delay of messages from source to destination, by smart single copy forwarding algorithms [8] and multiple copy forwarding algorithms [9]. In [9], they state that single copy forwarding is often a good starting point for the multiple copy forwarding, as it provides an upper bound for the average delay of messages. Therefore, it is worthwhile to analyze the delay of a message for single copy forwarding algorithm in MONs. Mobility-based forwarding [10] uses mobility information to select the intermediate mobile nodes which we refer to them as RNs. The selection of RNs is based on the current moving direction of the RN. The direction is estimated by finding the angle between an auxiliary vector from the current location of RN to destination and the velocity of the RN and then, the decision is basically made based on this angle. If the angle is less than $\pi/2$, RN is expected to move closer to the destination. This method assumes that each node knows its own location and the location of the destination.

Probability-based forwarding [11] forwards the message to a RN based on its probability of delivering the message to the destination. This probability is obtained by maintaining a prediction scheme which increases the probability if one frequently encounters the destined node or the probability decays with time from the last encounter. The Context-Aware Routing, introduced in [12], is similar to Probability-based forwarding, except it computes delivery probability based on the prediction of attributes in the context of a set of parameters related to the message delivery, like mobility patterns. The prediction is done by a Kalman filter.

All the above single copy forwarding schemes try to reduce the delay of messages, by utilizing extra information to make a better decision for selecting RNs that have a better delivery probability. However, they all fail to show the expected improvement in the delay by selecting mobile nodes with a better delivery probability for the message relay. Spyropoulos [8] has divided the single copy message routing in MONs into four distinctive categories, which are good representatives of different routing approaches: direct transmission [4], randomized routing, utility-based routing with 1 hop diffusion, and utility-based routing with transitivity. Each routing method employs only the last encounter time of each RN, as the only source of information to make a decision on whether to forward the message to the next node or not. Each category of routing schemes is analyzed and its expected delay is provided with respect to the nodes' transmission range. Even though the results give expected delay between a mobile source and a mobile destination, their results are still direct function of expected hitting time of a random walk. However, their results are limited to these four categories of routing algorithms and can not be generalized any further. We model this decision problem of selecting the mobile nodes with better delivery probability as a biased random walk of a message with different bias level $\alpha$. This allows all single copy forwarding algorithms to be represented by a single variable (bias level).

Beraldi published a work related to finding the hitting time of biased random walks for opportunistic searching problems in wireless networks [7]. He derived an expression for the delay (hitting time) of a biased random walk in uniform wireless networks based on the relative movement of biased random walks. But, it was done under a continuous region so the final expressions are very different from ours, since our results are from the biased random walk in finite tilted square graph. Even though, we both arrive at the same conclusion as the hitting time is very sensitive to bias level, here we provide a richer analysis about the effects of bias level.

## III. Problem Formulation

First, we define the topology of MON on Manhattan area and the movement of the messages carried by RNs. Then derive an exact expression, approximation, and bound of the delay of messages in MON on Manhattan area.

### A. Network Topology and Mobility Model

Suppose, RNs are moving at a constant speed and independently in the streets of Manhattan and all messages are generated at the intersection point of the streets (or messages are forwarded to nearest intersection points through multiple hop communication). The devices at intersection points store the messages until the RN enters its communication range and messages will be carried by RNs until they reach the sink node. We assume a sink node (destination of all messages) is located at the center of the Manhattan such that the chance for RNs to encounter the sink can be maximized and the messages in any RNs can be forwarded to another RN if they are both at the same intersection point. The forwarding decisions are made from a single copy message forwarding algorithm. For example, when a RN with messages encounters another RN, the algorithm forwards the message to other RN if it has higher

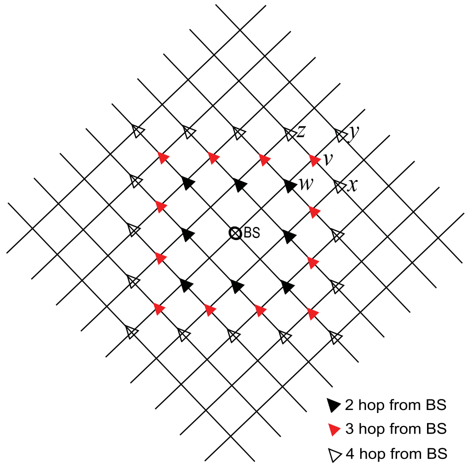


Fig. 1. The grid network with the sink (BS) at the center is showing the vertices that are equal hops away with the same shape and color.

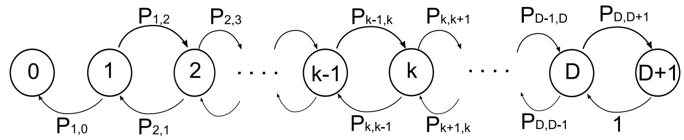


Fig. 2. Markov chain $G^1$ model representing the motion of a message relative to the sink location. The state 0 denotes the sink location where messages will be absorbed and state $D+1$ is boundary of $G$ where the message will be pushed back to state $D$ in its next transition

tendency to move closer to the sink location. This tendency can be predicted for the near future and is usually maintained by each RN. Here, we are not concerned with neither the prediction method nor the maintenance, but simply assume that there is an oracle which makes it available at the time of the forwarding process, which is similar to an oracle assumption in section 4 of [7]. In a single copy message forwarding, the messages do not get duplicated when they are forwarded to other RNs. Therefore, the expected delay of the message depends less on the number of RNs in the network but more on forwarding algorithm.

Regardless of what algorithm is used, any smart forwarding algorithm would forward the message to the RN that is likely to get closer to sink and remain close to sink to increase the chance of hitting the sink, or other RNs that will encounter the sink in the near future. So, from the message point of view, it's mobility pattern follows a biased random walk in the Manhattan area and its bias level depends on the performance of smart forwarding algorithms, number of RNs in the network. The direct modelling of every smart forwarding algorithms often hides it asymptotic effects on the average delay of messages. Therefore, we study the average delay of a message on the MON in Manhattan area by analyzing the hitting time of a biased random walk at its bias level $\alpha$.

Let, a tilted square grid graph $G = (V, E)$ represents the Manhattan streets, where $V = \{v_1, v_2, \cdots, v_{n-1}, o\}$ is set of nodes representing the intersection points in Manhattan and $o$ is the sink node as shown as Fig. 1. The square grid graph (or square lattice graph) is often used to approximate the Manhattan area to study the effects of mobility on the performance of routing protocols for Mobile Ad-hoc Networks [13]. Let, $N_i$ denotes a set of neighbors of node $v_i$. The edge transition probabilities for the biased random walk at $v_i \in V$ is determined by the degree of $v_i$, $\rho(v_i)$ and the number of neighbors of $v_i$ which helps the random walk move closer to the sink node $o$, which is denoted as $|W_i|$.

In the random walk, the probability that the walk at $v_i$ moves closer to the sink after moving one step, which is denoted as $P(d(v_j, o) < d(v_i, o)|v_j \in N_i)$, is $\frac{|W_i|}{\rho(v_i)}$. In a biased random walk, if $\alpha$ helps to increase this probability by taking a fraction of probability from each edge that moves away from the sink and adds them back to the edges that are moving closer to the sink, then it can be formally defined as,

$$P_\alpha(d(v_j, o) < d(v_i, o)|v_j \in N_i) = \frac{|W_i| + \alpha\,(\rho(v_i) - |W_i|)}{\rho(v_i)}. \tag{1}$$

Then, the edge transition probability of edge $(v_i, v_j)$ is simply,

$$p_{i,j} = \begin{cases} \frac{P_\alpha(d(v_j,o) < d(v_i,o)|v_j \in N_i)}{|W_i|} & \text{, if } v_j \in W_i \\ \frac{1 - P_\alpha(d(v_j,o) < d(v_i,o)|v_j \in N_i)}{|N_i| - |W_i|} & \text{, if } v_j \notin W_i. \end{cases} \tag{2}$$

When $\alpha = 0$, the $P_\alpha(d(v_j, o) < d(v_i, o)|v_j \in N_i) = \frac{|W_i|}{\rho(v_i)}$. This is equivalent to the case of unbiased random walks where a walk can move to any of its adjacent vertices with equal probability. When $0 < \alpha < 1$, the message is likely to move closer to the sink at each stage of the random walk. It is obvious that if $\alpha = 1$, the message will always move towards the target. If the message has started from node $v_i$ which $d(v_i, o) = d$ hops away from $o$, it will reach the target after exactly $d$ such transitions. For our problem, we assume that $0 \le \alpha \le 1$ since any rational message forwarding algorithm would do at least better than an unbiased random walk.

### B. Mapping from 2-D Random walks to 1-D Random walks

Given that the sink node $o$ is at the center of the tilted grid $G$, one can view the movement of the biased random walk in $G$ relative to a location $o$. The relative motion of the biased random walk after one transition is either moving one hop closer to the sink node or moving one hop away from the sink node. Based on this observation, we can map the biased random walk in $G$ to a biased random walk in a birth-and-death like Markov chain, $G^1$, where each state represents a rectilinear distance from the sink to other nodes $v_i \in V \backslash o$, $d(o, v_i)$, as shown as Fig. 2.

The state transition probabilities, $P_{i,j}$, are derived from Eqn.1. Let, a set $V_k$ represents a group of nodes that are exactly $k$-hops away from the sink node $o$ and let $max\{k\} = D + 1$. In the tilted square grid, the number of nodes exactly $k$-hop away from the sink node is $|V_k| = 4k$. There are exactly four vertices in $V_k$, which has three outgoing edges moving 1-hop away from the sink and one edge moving 1-hop closer(in) to the sink, which are denoted as $O_{k,3} \subset V_k$ and $I_{k,1} \subset V_k$

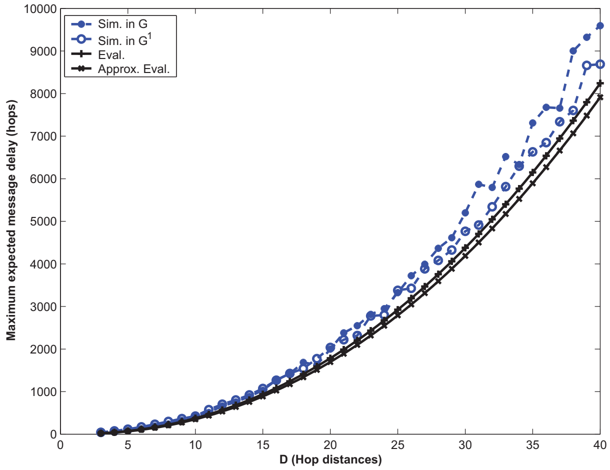


Fig. 3. The simulation shows that the expected delay of a random walk starting at the boundary of tilted grid increase quickly as the radius of tilted grid increase from 3 to 40. The graph also show that expected hitting time of random walk in Markov chain $G^1$ closely matches with result from tilted square grid $G$. It also show exact expression of average delay and its approximation follows closely with simulation results.

respectively. The rest of them has two outgoing edges moving 1-hop away from the sink and two moving closer to the sink, which are denoted as $O_{k,2} \subset V_k$ and $I_{k,2} \subset V_k$ respectively. For example, if a message is at $v$ the edge set $\{(v,x),(v,y),(v,z)\}$ increases distance by one hop and only edge $(v,w)$ reduces distance by one hop from center, as shown as Fig. 1. Assume all RNs are at its stationary distribution, then the stationary probability of a biased random walk being in the set $O_{k,3}$ is $\frac{4}{4k}$and the stationary probability of a biased random walk being in the set $O_{k,2}$ is $\frac{4k-4}{4k}$. Since, each outgoing edge is chosen by the RN with an equal probability of $1/4$ in the case of unbiased random walk, the state transition probabilities are,

$$\begin{aligned} P_{k,k-1} &= \frac{1+\alpha}{2} + \frac{\alpha-1}{4k}, \\ P_{k,k+1} &= \frac{1-\alpha}{2} + \frac{1-\alpha}{4k}, \\ P_{0,1} &= 0, \\ P_{D+1,D} &= 1. \end{aligned} \tag{3}$$

The $P_{0,1} = 0$ since state 0 is an absorption state which means message has reach the sink. State $D+1$ represents the message being in the boundary of tilted grid $G$.

## IV. Average Message Delay Analysis

Since the transmission delay is not significant as compared to the delay from RNs carrying the message, our delay analysis focuses on the time spent by the message in RNs before it finally reaches the sink. We refer to this delay as average message delay. This delay is proportional to hitting time of the biased random walk in $G^1$ since the speed of RNs are assume to be constant.

### *A. Average Message Delay for Unbiased Random Walk*

First, we derive the worst average message delay for the message originated $d$ hops away from the sink which is the hitting time of unbiased random walk when $\alpha = 0$. The state transition probability of unbiased random walk in Markov chain $G^1$ are,

$$\begin{aligned} p_{k,k+1} &= \frac{3}{4}\frac{1}{k} + \frac{1}{2}\frac{k-1}{k} = \frac{2k+1}{4k} \\ p_{k,k-1} &= 1 - p_{k,k+1}, \end{aligned}$$

for $0 < k < D$.

Instead of solving the Markov chain directly for an expected hitting time of the random walk starting at state $d$, we have follow the method using special structure of this Markov chain from Ross's book [14] which makes good use of the Markov chains property to enter state $(k-1)$ for the first time, the random walk must have entered the state $k \le D$. Let $N_i$ denotes the number of additional transitions that it takes the chain when it first enter the state $i$ until it transits to state $i-1$. Then, the expectation of $N_i$, $E[N_i] = \mu_i$, represented as a recursive function,

$$\begin{aligned} \mu_i &= 1 + E\left[N_{i+1} + N_i\right] p_{i,i+1} \\ &= \tfrac{4i}{2i-1} + \tfrac{2i+1}{2i-1}\mu_{i+1}. \end{aligned} \tag{4}$$

A generalized formula in a non-recursive form for $\mu_i$ is obtained when Eqn. 4 was solved reverse inductively from state $(D+1)$ as:

$$\mu_i = \frac{2(D+1)^2 - 2i^2 + 2i - 1}{2i-1}. \tag{5}$$

Then, average time taken for an unbiased random walk starting from state $d$ to reach state 0 for a first time, also known as hitting time, is $E\left[N_{d,0}\right] = \sum_{i=1}^{d} \mu_i$ and an exact expression and closed form approximation formula for $E[N_{d,0}$ is derived after extensive simplification using small mathematical tricks like $-2i^2 = -4i^2/2 = -((2i-1)(2i+1)+1)/2$.

$$E\left[N_{d,0}\right] = \left(2(D+1)^2 - \frac{1}{2}\right)\sum_{i=1}^{d}\frac{1}{2i-1} - \frac{d^2}{2} \tag{6}$$

$$\approx \left(2(D+1)^2 - \frac{1}{2}\right)\left(\ln\left(2d-1\right) + \gamma + \varepsilon_{2d-1}\right) - \frac{d^2}{2} \tag{7}$$

$$= O\left(D^2 \log d\right), \tag{8}$$

where $\gamma$ is Euler-Mascheroni constant and

$$\lim_{(2d-1)\to\infty} \varepsilon_{2d-1} \to 0.$$

In Eqn. 6, the summation term is approximated by

$$\sum_{i=1}^{d} \frac{1}{2i-1} \approx \frac{1}{2}\sum_{i=1}^{2d-1}\frac{1}{i}$$

and logarithmic result is due to Leonhard Euler's rate of divergence of Harmonic series.

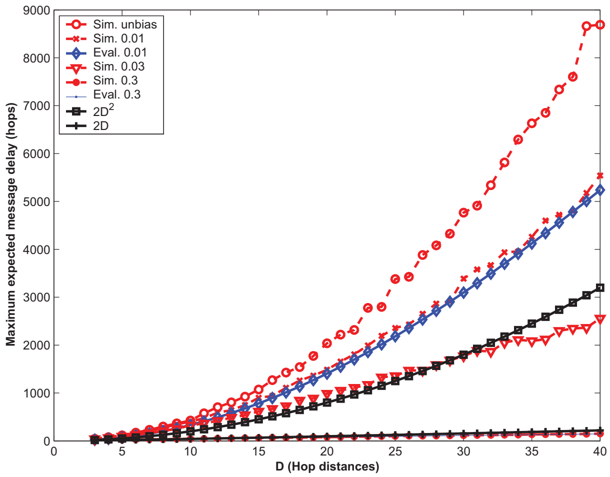


Fig. 4. The simulation shows that the expected delay of a biased random walk starting at the boundary of tilted grid reduces quickly as the bias level increase from 0 to 0.01 and the delay become almost linear when bias level is 0.03. We also show that when bias level is 0.3 the expected delay is significantly reduced and is completely linear. This result supports our Theorem4.1 and Corrollary 4.2

From Fig. 3, we show that the mapping of a unbiased random walk in tilted grid $G$ to 1-D Markov chain $G^1$ well preserves all the characteristics of unbiased random walk in $G$. From simulation, we also show that the exact formulation and approximation match closely to the simulation results for the case of unbiased message forwarding. It is worth highlighting that our result is not just the worst case of message delay where a message starting from state $d = D+1$ but it also allow us to calculate any average delays for messages originated from any vertex $v \in V$.

**Remark**: The number of vertices in the grid of Fig. 1 is $n = |V| = O(D^2)$. So, $O(n \log n) = O\left(D^2 \log D\right)$. The $O(n \log n)$ is a well known result of a worst hitting time for a random walk in 2-dimensional grid graph (in Table 1 of [15]). As a consequence, the Eq. 8 is also equal to a bound for the hitting time for a random walk in 2-dimensional grid graph. This means our problem modelling and our analysis could be used to approximate the hitting time in grid topology.

### B. Average Message Delay for biased Random Walk

In this section, we obtain the average message delay of biased random walk by following a similar procedure used in an unbiased random walk case. First, we find the recursive formula for $\mu_k$,

$$\begin{aligned}\mu_k &= 1 + P_{k,k-1}\left(\mu_{k+1} + \mu_k\right) \\ &= \frac{1}{P_{k,k-1}} + \frac{P_{k,k+1}}{P_{k,k-1}}\mu_{k+1}.\end{aligned} \tag{9}$$

$\mu_{D+1} = 1$ since state $D+1$ is last state and $P_{D+1,D} = 1$. We obtain preceding non-recursive equation from Eq. 9 as:

$$\mu_j = \frac{1}{P_{j,j-1}}\left(1 + \sum_{i=j}^{D-1}\prod_{l=j}^{i}\frac{P_{l,l+1}}{P_{l+1,l}}\right) + \prod_{l=j}^{D}\frac{P_{l,l+1}}{P_{l,l-1}}. \tag{10}$$

We have numerically computed the $E_\alpha\left[N_{D+1,0}\right]$ at different $D$ by taking summation of $\mu_i$, for $i = 1:D$, and then compared it to simulation as shown as Fig. 4. However, in order to see the impact of the bias level $\alpha$, we further simplify the $E_\alpha\left[N_{D+1,0}\right]$ after substituting Eqn. 10 to obtain following Theorem. We have abused the notation of bias level $\alpha$ by additional variable $x$ as subscript to indicate, the bias level is not necessarily constant but it can actually vary with current location of biased random walk.

*Theorem 4.1:* A biased random walk with bias level $0 \le \alpha \le 1$ in $G$, the hitting time, $E_{\alpha_x}\left[N_{d,0}\right]$, for walk starting at $d$ hop away from the sink has the following bounds:

$$E_{\alpha_x}\left[N_{d,0}\right] = \begin{cases} O\left(D^2 \log d\right) & , for \quad \alpha_x = 0 \\ O\left(D \log d\right) & , for \quad 0 < \alpha_x < \frac{1}{2x+1} \\ O\left(\max\{d, D-d\}\right) & , for \quad \alpha_x = \frac{1}{2x+1} \\ O(d) & , for \quad \frac{1}{2x+1} < \alpha_x \le 1, \end{cases}$$

where $x \in X$ and $X = \{d, \ldots, 0\}$ is a series of distances of a biased random walk relative to the sink during its trip from the origin of walk $d$ to sink $o$. $\alpha_x$ is denoted as the bias level of the random walk at $x \in X$.

*Proof:* Since the proof of Theorem 4.1 is technical and lengthy, deferred to a technical report [16]. ■

Next, we provide a detailed discussion about the Theorem 4.1.

### C. Discussion on Bound for the Average Message Delay with respect to the Bias Level

We have introduced a method of numerically computing the hitting time of biased random walks starting anywhere at the intersection of a tilted square grid and using this exact expression we derived the upper bound of average message delay. Since we assume that each step of the transition is of unit length, the message delay following a biased random walk in MON on Manhattan area is directly proportional to the result obtained in Theorem 4.1. It also concludes that the average delay of a message is very sensitive to the bias level. This result is supported by our simulation as shown as Fig. 4. A similar conclusion was also obtained in [7] for a biased random walk in Uniform Wireless Network. However, Theorem 4.1 reveals another very important fact that the average message delay is sensitive to bias level of a random walk at its current location.

To elaborate this fact, if the smart forwarding in MONs can maintain the bias level of random walking at $x$ larger than $1/(2x+1)$, the average delay would be $O(d)$. Theorem 4.1 shows when messages originated far from the sink, only small increase in its bias level is enough to maintain the average delay to be $O(d)$ since $\alpha_x$ is inversely proportional to $x$. This mean even if the performance of message forwarding algorithm is poor (e.g. due to inaccurate prediction of RNs future positions), it can still bring messages closer to sink location in linear time. When the messages get closer to sink the information about the sink location also get richer and more accurate. This result explains why most of the smart (not very smart) forwarding algorithm, listed in the section II, produces good performance in the message delay. Further more, the Theorem 4.1 also proof following corollary.

*Corollary 4.2:* If the bias level of a message can be maintained higher than $1/3$ for its entire trip to sink due to whatever reasons, The worst average message delay would be $O(d)$.

For example, this corollary indicates, it is not necessary to forward the message to any other intermediate RNs with higher tendency to move closer to sink if the current RN is carrying the message with bias level higher than 1/3, since this will not change the order of average message delay. It is a useful information to be considered while designing the routing algorithm which optimizes delay of a message as well as energy and interference in MONs. However, measuring this bias level is still an open problem in practice.

When the movement of the message is unbiased, $\alpha_x = 0$, the expected message delay can be as large as $O(D^2)$, even if the message originated very close to the sink as long as $d > 1$. When $d = 1$, $E_{\alpha_x}[N_{d=1,0}] = 0$. This means the message can directly be forwarded to the sink in constant time. So, intuitively, when the messages are just one hop away from the sink the device at these intersection points would keep the messages until it encounters a RN that is moving towards the sink. Therefore, the message delay would be only one transition delay. In this paper, we set the worst case mobility of a message as an unbiased random walk ($\alpha = 0$). However, we can still numerically compute the average message delay for the case when $\alpha < 0$ using Eqn. 10 and state transition probabilities in Eqn. 3.

## V. Conclusion and Future Works

We have studied the average message delay of single copy forwarding in MONs on a Manhattan area, by modelling the movement of message forwarding as a biased random walk on the tilted square grid graph. Since, it is difficult to directly analyze the delay in the tilted square grid we have grouped the vertices with equal rectilinear distance to the sink as one set and view the movement of message starting at hop distance $d$ relative to location of the sink at the center of grid. This relative movement of a message allow us to map the biased random walk in tilted square grid to a biased random walk in 1-D Markov chain, while preserving the biased random walk characteristics in 2-D. Then, by using Markov properties, we derive an exact expression for hitting time of biased random walk which represents an average delay of message till it reaches the sink. We also provide closed form approximation of the hitting time expression for the case of unbiased random walks (used to indicate the worst case delay of a messages in MON). For biased random walks, we have derived an upper bound for the average delay of a message to explain the effect of bias level.

Finally, this upper bounded results show that the average delay of a message is very sensitive to the bias level. Moreover, it is sensitive to the bias level at each corresponding location of the trip to the sink. We also find out that if a walk can maintain its bias level greater than a constant threshold of $1/3$ then the average delay of a message can be kept linear to a rectilinear distance from the origin of a message to the sink node. All these theoretical conclusion were supported by the simulations.

As for our future works, we will model various existing single copy routings as a biased random walk and analyze their corresponding bias level to compare their performance and explain their limitations. Also, we will look at the methods of measuring the bias level in practice.

## References


[1] L. Pelusi, A. Passarella, and M. Conti, "Opportunistic networking: data forwarding in disconnected mobile ad hoc networks," *IEEE Communications Magazine*, vol. 44, no. 11, p. 134, 2006.

[2] A. Balasubramanian, B. Levine, and A. Venkataramani, "DTN routing as a resource allocation problem," in *Proceedings of the 2007 conference on Applications, technologies, architectures, and protocols for computer communications*, 2007, p. 384.

[3] W. Zhao, M. Ammar, and E. Zegura, "A message ferrying approach for data delivery in sparse mobile ad hoc networks," in *MobiHoc '04: Proceedings of the 5th ACM international symposium on Mobile ad hoc networking and computing*. New York, NY, USA: ACM, 2004, pp. 187–198.

[4] M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, no. 4, pp. 477–486, August 2002.

[5] P. Gupta and P. Kumar, "The capacity of wireless networks," *Information Theory, IEEE Transactions on*, vol. 46, no. 2, pp. 388–404, Mar 2000.

[6] S. Basagni, A. Carosi, E. Melachrinoudis, C. Petrioli, and Z. M. Wang, "Controlled sink mobility for prolonging wireless sensor networks lifetime," *Wirel. Netw.*, vol. 14, no. 6, pp. 831–858, 2008.

[7] R. Beraldi, "Biased random walks in uniform wireless networks," *Mobile Computing, IEEE Transactions on Mobile Computing*, vol. 8, no. 4, pp. 500–513, April 2009.

[8] T. Spyropoulos, K. Psounis, and S. C. Raghavendra, "Efficient routing in intermittently connected mobile networks: the single-copy case," *IEEE/ACM Trans. Netw.*, vol. 16, no. 1, pp. 63–76, 2008.

[9] T. Spyropoulos, K. Psounis, and C. S. Raghavendra, "Efficient routing in intermittently connected mobile networks: the multiple-copy case," *IEEE/ACM Trans. Netw.*, vol. 16, no. 1, pp. 77–90, 2008.

[10] J. LeBrun, C.-N. Chuah, D. Ghosal, and M. Zhang, "Knowledge-based opportunistic forwarding in vehicular wireless ad hoc networks," in *Vehicular Technology Conference, 2005. VTC 2005-Spring. 2005 IEEE 61st*, vol. 4, May-1 June 2005, pp. 2289–2293 Vol. 4.

[11] A. Lindgren and A. Doria, "Probabilistic routing protocol for intermittently connected networks," in *draft-lindgren-dtnrg-prophet-02.txt*, August 2006.

[12] M. Musolesi, S. Hailes, and C. Mascolo, "Adaptive routing for intermittently connected mobile ad hoc networks," *WoWMoM Sixth IEEE International Symposium*, pp. 183–189, 2005.

[13] F. Bai, N. Sadagopan, S. Member, S. Member, B. Krishnamachari, and A. Helmy, "Modeling path duration distributions in manets and their impact on reactive routing protocols," *IEEE Journal on Selected Areas in Communications*, vol. 22, pp. 1357–1373, 2004.

[14] S. M. Ross, *Introduction to Probability Models, Seventh Edition*. San Diego, CA, USA: Academic Press, Inc., 2000.

[15] N. Alon, C. Avin, M. Koucky, G. Kozma, Z. Lotker, and M. R. Tuttle, "Many random walks are faster than one," in *SPAA '08: Proceedings of the twentieth annual symposium on Parallelism in algorithms and architectures*. New York, NY, USA: ACM, 2008, pp. 119–128.

[16] J.-H. Jun, "Expected First Time to Hit Center of Finite Tilted Square Grid for a Bias Random Walk," University of Cincinnati, Department of Computer Science and and Informatics, Tech. Rep., 01 2010, also available at http://junjunghyun.wordpress.com/research/publications/conferences.