Reliable Broadcast Transmission in Wireless Networks Based on Network Coding

Weiwei Fang, Feng Liu, Zhen Liu
School of Computer and Information Technology
Beijing Jiaotong University
Beijing, P.R.China
{wwfang, fliu, zhliu}@bjtu.edu.cn

Lei Shu, Shojiro Nishio
Nishio Lab, Department of Multimedia Engineering
Osaka University
Osaka, Japan
{lei.shu, nishio}@ist.osaka-u.ac.jp

Abstract—**Recently, XOR based network coding has been applied to the loss recovery of reliable broadcast transmission in wireless networks, where the source can XOR multiple lost packets with distinct receivers together into one packet and transmits this combined packet in one retransmission, resulting in a significant improvement on transmission efficiency. The problem of finding the optimal XOR coding set that minimizes the overall number of transmissions for loss recovery has been proved to be NP-hard. In this paper, we propose an efficient heuristic algorithm based on vertex coloring for giving an approximately optimal solution to address the problem, and then apply this heuristic algorithm into the existing XOR-based retransmission schemes. Simulation results are given to demonstrate superior performance of our algorithm over previously proposed works.**

Keywords-wireless networks; network coding; broadcast; retransmission; vertex coloring

I. INTRODUCTION

Broadcast is an important mechanism in wireless networks for disseminating identical data information in a one-to-many fashion. It has been widely employed in quite a many network applications like content delivery and device configuration. It's required that all the information sent by the source must be correctly received by every receiver. However, wireless links are more error-prone than their wired counterparts. Therefore, it is necessary to use some appropriate error-control schemes to provide reliability guarantees. So far, the mostly widely applied technique for handling packet losses in the network transmission is Automatic Repeat reQuest (ARQ), which uses timeouts and acknowledgements for the loss recovery. Though ARQ is very efficient in the unicast communication scenario, it is not so efficient in the broadcast transmissions since the original source needs to rebroadcast the packet even if only one receiver has not received that packet.

Network coding, which was originally proposed in information theory [1], has become a promising approach to improve throughput or save bandwidth in wireless networks. The core idea is to allow the sender to appropriately mix multiple data by the XOR operation [2] or a linear combination [3] before sending these coded data to the receivers. Generally, it is more preferable to use XOR coding in the implementation [2] since both encoding and decoding operations are much simpler than the linear network coding technique. Data packet retransmission based on XOR coding for wireless broadcast is a recent field of study proposed by Nguyen et al. [4][5]. The main idea is to enable the sender to combine multiple lost packets by XOR-ing and transmit this XOR-ed packet to multiple distinct receivers. Upon receiving a XOR-ed packet, a receiver is able to recover its lost packet by XOR-ing this XOR-ed packet with certain packets that it has already received previously. As such, one retransmission can enable multiple receivers to recover the packets they lost, thus the number of retransmissions can be effectively reduced while the wireless bandwidth can be efficiently utilized. This was soon followed by a rich body of research work. Zheng and Sinha [6] proposed a XOR-based approach to optimize the utilization of limited retransmission buffer space in presence of random burst packet losses. Ghaderi et al. [7] presented an analytical work on reliability performance of XOR coding compared with ARQ and FEC (Forward Error Correction) in the lossy wireless network. Li et al. [8] introduced timing control into XOR coding to further enhance the network performance by adaptively adjusting node transmission behavior on the basis of the network traffic conditions. Wang et al. [9] try to make XOR coding applicable and efficient for loss recovery in vehicular safety communication scenarios by optimizing the number of packets to be combined on the basis of the sender-receiver distance. Some network test-beds have already been constructed to evaluate the performance of XOR-based coding for loss recovery in real wireless systems, such as IEEE 802.11 [2] and IEEE 802.16 [10].

However, so far little work has been devoted to optimal XOR decision, which decides the coding set of the lost packets with distinct intended receivers for XOR-ing. The optimization objective is to minimize the overall number of transmissions for loss recovery so as to improve the transmission efficiency as much as possible. However, in their pioneer work Nguyen et al. didn't present any solution for this problem [4][5]. This problem has been proved as a complex NP-complete problem in [11][12]. Recently, a few of heuristic algorithms was proposed in literature [13][14][15]. Unfortunately, most of these proposals have their own drawbacks and limitations (which will be explained later in detail in Section II), making them impractical for general use. To address this challenge, we propose in this paper a novel vertex coloring-based heuristic algorithm, which can be applied in the XOR coding-based schemes to further guarantee transmission efficiency in lossy networks.

The organization of our paper is as follows. We first discuss the research motivation and a few of related works in Section II. Section III presents design details of the proposed algorithm. The simulation and discussion are given in Section IV. Finally, the paper is concluded in Section V.

II. MOTIVATION

In general, reliable broadcast doesn't allow data loss but can tolerate delay caused by loss recovery. In traditional retransmission based methods, the sender simply retransmits the lost native packets one by one. Recently, XOR-based network coding was applied by Nguyen et al. [4][5] to reliable broadcast in wireless networks. This work can be described as follows: The sender S broadcasts a fixed number of n native packets one by one to m (m>1) receivers, and obtains the results on packet losses of these receivers through positive or negative acknowledgements (i.e., ACK or NACK). The receiving results are maintained at the sender as a $m\times n$ table T in which each entry $t_{i,j}$ ($1\leq i\leq m$, $1\leq j\leq n$) is used to indicate whether the receiver r_i has received packet P_j or not, i.e., $t_{i,j}$=0 if P_j is correctly received by r_i and $t_{i,j}$=1 otherwise. Rather than retransmitting the lost packets one by one to the intended receivers, the sender tries to find a set of lost native packets without common intended receivers to XOR and transmits the XOR-ed packet. A receiver is able to recover its lost packet by XOR-ing this XOR-ed packet with some packets that it has already received previously. In this way, the sender keeps sending out XOR-ed packets until no packet is found left from the packet-loss table, and then starts the transmission of next n packets. There are two specific schemes for coding operations: the static one generates all the XOR-ed packets for once at the beginning of the retransmission phase and will repeatedly retransmit the same XOR-ed packet until all its intended receivers receive this packet successfully, while the dynamic one updates the XOR-ed packets in each retransmission according to the latest results in the packet-loss table for a further improvement on transmission efficiency.

Fig. 1 shows an example of the packet-loss table, which represents the receiving states of five receivers r_1 ~ r_5 for packets P_1 ~ P_6. Traditionally, each one of the six packets is retransmitted alone and sequentially by the sender until successfully received by all the five receivers. When using the static scheme, the sender would XOR P_1 and P_3 together to packet P ($P=P_1\oplus P_3$) and broadcast packet P. Once r_1, r_2 or r_5 receivers P, any one of them can recover P_1 by $P_1=P\oplus P_3$. Similarly, r_3 or r_4 can recover P_3 by $P_3=P\oplus P_1$. The sender will keep transmitting P until it has been successfully received by all the receivers, and then start the transmission of next XOR-ed packet, e.g., $P'=P_2\oplus P_4$. When using the dynamic scheme, however, the sender would generate a new XOR-ed packet according to the latest receiving results updated by received acknowledgements. For example, suppose packet P is received by r_1 and r_2 but not by r_5 after the transmission, the sender can XOR P_1, P_5 and P_6 together for next transmission.

By XOR-ing lost packets together in this way to transmit for loss recovery, the average number of intended receivers per packet can be increased while the number of total retransmissions can be reduced.

	P_1	P_2	P_3	P_4	P_5	P_6
r_1	1	0	0	1	1	0
r_2	1	1	0	0	1	0
r_3	0	1	1	0	1	0
r_4	0	0	1	1	0	1
r_5	1	1	0	0	0	0

Fig. 1 An example of packet-loss table

To improve transmission efficiency as much as possible, it is of great necessity for the sender to find the optimal way to combine the lost packets so that the total number of transmissions can be minimized, and each of the receivers is able to decode all the lost packets it needs. As for the packet-loss table in Fig. 1, the optimal encoding set is found unique as $\{P_1\oplus P_3, P_2\oplus P_4, P_5\oplus P_6\}$. However, Nguyen et al. [4][5] paid little attention to the solution for this problem. NCWBR [13] used a heuristic method by searching the first entry equal to 1 in each row of the table and combining the corresponding packets together by XOR-ing. This approach can not always guarantee to generate decodable XOR-ed packet in each time of transmission. Take Fig. 1 for an example, NCWBR will firstly XOR P_1, P_2, P_3 together, which makes r_2, r_3 and r_5 not be able decode this XOR-ed packet. Therefore, a lot more transmissions are needed. Actually, NCWBR can not fully exploit the potential coding opportunities for improving the bandwidth efficiency. Another approach proposed by Kuo et al. [14] employed reception estimation to determine how to generate the coding sets. However, this approach was complex to use and might not be accurate enough. Wang et al. proposed an encoding decision algorithm [15] for data dissemination in wireless sensor networks, which takes both link unreliability and sleep scheduling into consideration. This algorithm wasn't suitable for wireless networks that are always maintaining in the working state. In short, an efficient heuristic algorithm is demanded in order to find a proper solution for the XOR decision problem.

III. DESIGN DETAILS

A. Problem Formulation

We consider a one-hop wireless network with a single sender S and a set of m receivers $R=\{r_1, r_2, \ldots, r_m\}$. The sender needs to transmit a set $P=\{P_1, P_2, \ldots, P_n\}$ of packets to the receivers. Due to packet losses, each receiver r_i requires a certain subset packets in P, $Sr_i=\{P_j|t_{i,j}=1, 1\leq j\leq n\}$, to be retransmitted by the sender. To reduce transmissions as much as possible, our goal is to find the set of XOR-ed packets with minimal cardinality that allow each receiver to recover the lost packets it wants. Let this set denoted by X, and the cardinality of X is denoted by $|X|$.

We can formally formulate optimal XOR decision problem as follows:

Given: $T_{m\times n}$
Variables: $1\leq i\leq m$, $1\leq j\leq n$, $1\leq k\leq |X|$
Encode packet: $x_k=a_{k,1}P_1\oplus\ldots\oplus a_{k,n}P_n$, $a_{k,j}\in\{0,1\}$

Minimize: $|X=\{x_k\}|$
Subject to:

(1) if $\sum_{i=1}^{m} t_{i,j} = 0$, then $\sum_{k=1}^{|X|} a_{k,j} = 0$;

(2) if $\sum_{i=1}^{m} t_{i,j} > 0$, then $\sum_{k=1}^{|X|} a_{k,j} = 1$;

(3) $\sum_{j=1}^{n} a_{k,j} t_{i,j} \le 1$.

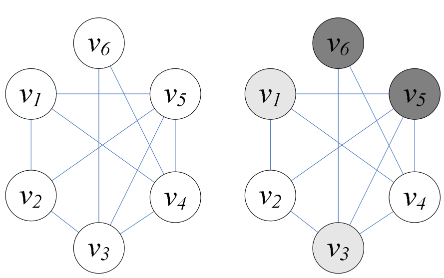
In the above formulation, constraint (1) ensures that the native packet which has been received by all receivers won't be combined into any XOR-ed packet, and constraint (2) ensures that the native packet which has not been received by at least one receiver will be combined into one of the XOR-ed packets. According to [2], any two lost packets that have common intended receivers should not be XOR-ed together; otherwise, the relevant receivers will not be able to decode the XOR-ed packet. Thus, constraint (3) is given as above.

B. *Vertex coloring-based heuristic algorithm*

Given the packet-loss table T, we can construct an undirected graph $G(V, E)$, where $V=\{v_l | \sum_{i=1}^{m} t_{i,l} > 0,\ 1 \le l \le n\}$ and $E=\{(v_l, v_{l'}) \in V^2 | l \ne l',\ \exists 1 \le i \le m,\ t_{i,l} \wedge t_{i,l'} = 1\}$. In graph G, each vertex v_l represents a lost packet P_l that is required by one or more receivers, while each edge $(v_l, v_{l'})$ denotes the mutual exclusiveness of the XOR operation between P_l and $P_{l'}$. Thus, our problem which aims to find the set of XOR-ed packets with minimal cardinality that allow each intended receiver to decode out one packet it lost can be transformed into coloring vertices in graph G with the chromatic number of colors.

For example, the packet-loss table in Fig. 1 is converted to the graph G in Fig. 2a, in which the packet P_i is represented by the vertex v_i. As illustrated in Fig. 2b, the optimal XOR set $\{P_1 \oplus P_3, P_2 \oplus P_4, P_5 \oplus P_6\}$ can be found by vertex coloring on graph G.

However, it has been proved that vertex coloring is also a NP-complete problem [16]. Since by now no polynomial-time algorithm is available to obtain the optimal solution of a NP-complete optimization problem, it is necessary to develop an efficient heuristic algorithm that has acceptable computational complexity, and is scalable to large number of receivers [12].



(a) (b)

Fig. 2 (a) The graph *G* for the packet-loss table in Fig.1, (b) the graph *G* colored with the chromatic number of colors

Heuristic algorithm for optimal XOR decision

Input: T
Output: X
Steps:

```
Construct graph G(V, E) corresponding to T.
Order the vertices in V by decreasing degrees.
while V≠Ø do
   Find a vertex v_m∈ V with the maximal degree.
   C={v_m}
   V=V \ v_m
   for each v∈ V do //traverse the vertices in order
      if ∀v'∈ C, (v, v')∉ E then
         C=C ⋃ {v'}
         V=V \ v'
      end if
   end for
   A new packet x is generated by XOR-ing the lost
   packets corresponding to the vertices in C.
   X= X ⋃ {x}
end while
return X
```

Fig. 3 Vertex coloring-based heuristic algorithm

In this work, we use the largest-degree first (LF) algorithm [17] for coloring approximation. The LF algorithm has been widely used in graph coloring researches. It tries to minimize the colors needed, but doesn't always yield a minimal coloring of the graph. Our algorithm is illustrated in Fig. 3.

We now show how the heuristic algorithm works on graph G in Fig. 2a. The sorted set of vertices is $\{v_3, v_4, v_5, v_1, v_2, v_6\}$. First, $v_m = v_3$ and $C=\{v_3, v_1\}$, so $x_1 = P_1 \oplus P_3$; Then, $v_m = v_4$ and $C=\{v_4, v_2\}$, so $x_2 = P_2 \oplus P_4$; Finally, $v_m = v_5$ and $C=\{v_5, v_6\}$, so $x_3 = P_5 \oplus P_6$. In all, $X=\{P_1 \oplus P_3, P_2 \oplus P_4, P_5 \oplus P_6\}$.

C. *Heuristic-based static and dynamic schemes*

By adopting the proposed heuristic algorithm above as the approximately optimal solution for the XOR decision problem, we can obtain the corresponding static scheme and dynamic scheme, which are shown in Fig. 4 and Fig. 5. It can be found that assuming no loss occurs in the transmission of XOR-ed packets, there will be no difference between the dynamic scheme and the static scheme. Actually the XOR-ed packets would be lost during transmission process, so the dynamic scheme can further reduce transmissions due to its dynamic update of the set of XOR-ed packets.

Heuristic-based static scheme

Steps:

```
Transmit n native packets one by one and build the
packet-loss table T.
Adopt the proposed heuristic algorithm to obtain the
XOR-ed packet set X.
for each x∈X do
   Repeatedly transmit the XOR-ed packet x until all its
   intended receivers receive it successfully.
end for
```

Fig. 4 Heuristic-based static scheme

Heuristic-based dynamic scheme

Steps:

1 Transmit n native packets one by one and build the packet-loss table T.
2 Adopt the proposed heuristic algorithm to obtain the XOR-ed packet set X.
3 **for** each $x \in X$ **do**
4   **if** x is received by all its intended receivers **then**
5   **continue**
6   **else**
7   Update T according to latest receiving results.
8   **goto** step 2
9   **end if**
10 **end for**

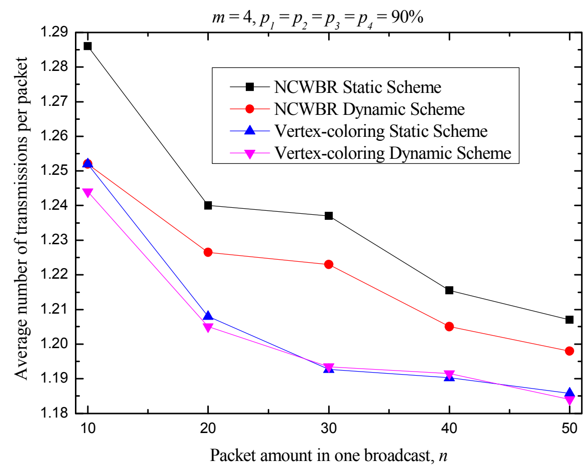
Fig. 5 Heuristic-based dynamic scheme

IV. PERFORMANCE EVALUATION

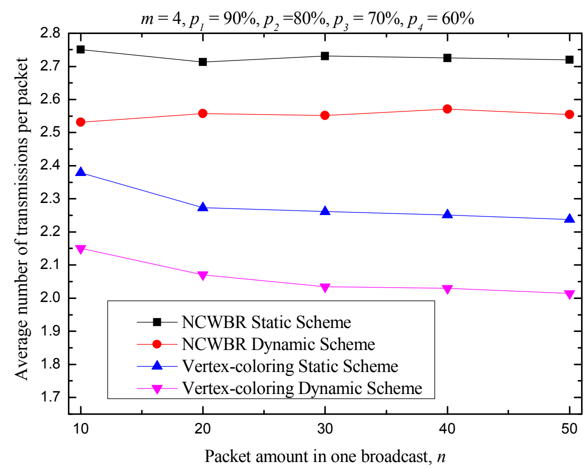
In this section, we investigate the transmission bandwidth (i.e., average number of transmissions per packet) of the two schemes (static/dynamic) with our algorithm and NCWBR under different sizes of the packet amount in one broadcast, different link-packet-loss probabilities and different numbers of receivers respectively.

In our simulation, packet loss at each receiver follows the Bernoulli distribution. In addition, packet losses at different receivers are uncorrelated. After the broadcast, each receiver has the chance to return ACK or NACK to the source node if necessary. Since ACK and NACK have very small size and very small loss probability, they are assumed to be instantaneous and lossless. For each scenario of parameter setting, our simulation conducts broadcast of $n \times 10^2$ packets. The packet delivery ratio of the link between S and receiver r_i is denoted by p_i in the result figures.

Fig. 6 illustrates the transmission bandwidth of different schemes versus the size of lost-packet buffer, in the cases of four receivers. From this figure, we can observe that in general the transmission bandwidth of the schemes with our algorithm decrease as the packet amount increases. When used in the same scheme, our algorithm is more bandwidth efficient than NCWBR: In low packet loss scenarios, the improvement of our algorithm over NCWBR is not obvious, only about 1.1% to 3.6%; While in high packet loss scenarios, our algorithm significantly outperforms NCWBR by reducing about 13.5% to 21.1% packet transmissions. This is because when the packet loss probabilities are very small, there are only a few of lost packets need to be retransmitted. Therefore, the chances for packet combination through XOR-ing are also very limited. We can also notice that in low packet loss scenarios, the differences of transmission performance between the static scheme and dynamic scheme are very small. The underlying reason is that the retransmission can always succeed in the first few attempts. When the packet loss rates are high, we can observe from Fig. 6b that the dynamic scheme significantly exceeds the static one no matter NCWBR or our algorithm is used. This verifies our previous analysis in subsection 3.3.



(a) The low packet loss environment



(b) The high packet loss environment

Fig. 6 Transmission bandwidth versus packet amount

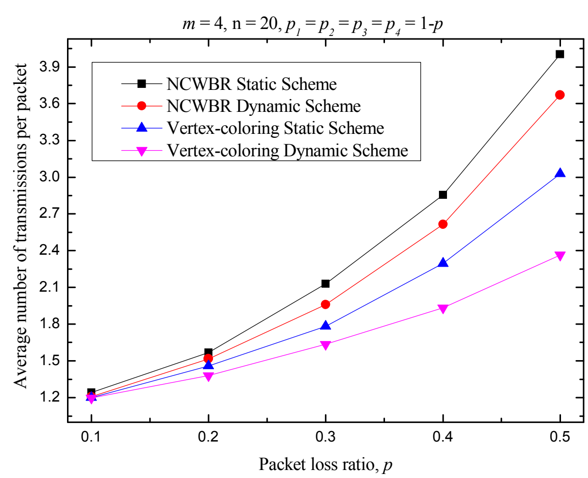
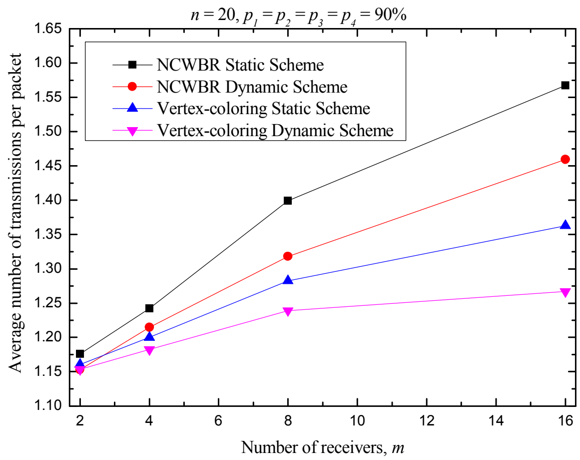
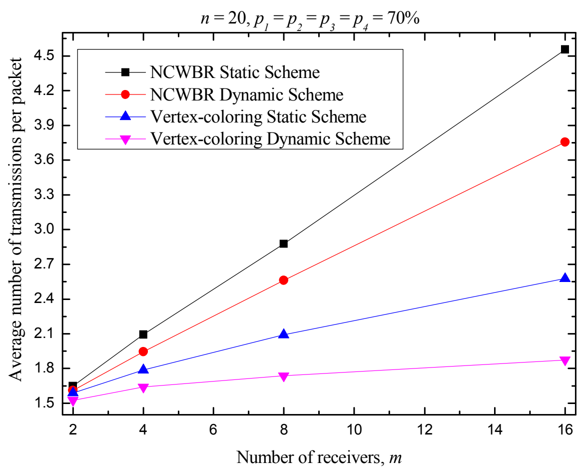


Fig. 7 Transmission bandwidth versus packet loss probability

Fig. 7 illustrates the transmission bandwidth of different schemes versus the packet loss probability p. When used in the static scheme, our algorithm further reduces the total transmissions of NCWBR by 3%, 7%, 16%, 20%, 24% respectively as p grows from 0.1 to 0.5. Meanwhile, when used in the dynamic scheme, our algorithm further reduces the total transmissions of NCWBR by 1%, 9%, 17%, 26%, 35%



(a) The low packet loss environment



(b) The high packet loss environment

Fig. 8 Transmission bandwidth versus number of receivers

respectively as p grows from 0.1 to 0.5. This observation implies that our algorithm is more capable to exploit coding opportunities to reduce overall transmissions than NCWBR, especially in high loss environments.

Fig. 8 illustrates the transmission bandwidth of different schemes versus different number of receivers. When there are only two broadcast receivers, our algorithm provides a little improvement to NCWBR. As m increases, the gap of the transmission bandwidth between NCWBR and our algorithm becomes much larger. We can even notice that our algorithm reduces about 50% transmissions of NCWBR when m=16. It is obvious that our algorithm is much more scalable to large number of broadcast receivers than NCWBR.

V. Conclusions

In this paper, we propose a novel vertex coloring based algorithm to solve the optimal XOR decision problem in the wireless network. The algorithm performance is evaluated through extensive simulations under different conditions. Simulation results show that our algorithm can not only achieve higher transmission efficiency (especially in the high packet loss environment), bust also be more scalable to larger number of broadcast receivers, as compared with NCWBR. Our future work will take packet delay into consideration and extend current work for delay-guaranteed reliable broadcast.

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