NAME:

CSCE 235 Quiz 1
February 2, 2007

Determine whether each of the following proofs is a direct proof, a proof by contraposition, or a proof by contradiction.

1. **Proposition.** If n is an integer and $n^3 + 5$ is odd, then n is even.

   Proof. Suppose that n is odd. Then there exists an integer k such that $n = 2k + 1$. So

   $$n^3 + 5 = (2k + 1)^3 + 5 = (8k^3 + 12k^2 + 6k + 1) + 5 = 8k^3 + 12k^2 + 6k + 6 = 2(4k^3 + 6k^2 + 3k + 3).$$

   Taking $\ell = 4k^3 + 6k^2 + 3k + 3$, which is an integer, we see that $n^3 + 5 = 2\ell$, so $n^3 + 5$ is even. Therefore, if $n^3 + 5$ is odd, then n is even. □

2. **Proposition.** There is no smallest positive rational number.

   Proof. Assume that there is a smallest positive rational number; call it r. Since r is rational, we may write it as $r = p/q$ for some integers p and q. Now consider the number $r/2 = p/(2q)$. Since r is positive, $r/2$ is positive, and $r/2$ is smaller than r. Moreover, since q is an integer, $2q$ is an integer. Thus $r/2$ is rational. This contradicts our assumption that r is the smallest positive rational number; therefore, we conclude that there is no smallest positive rational number. □

3. **Proposition.** Every odd integer is the difference of two perfect squares. [Recall that a *perfect square*, commonly called just a *square*, is an integer m with the property that there exists an integer k such that $m = k^2$.]

   Proof. Let x be an odd integer. Then $x = 2k + 1$ for some integer k. So

   $$(k + 1)^2 - k^2 = (k^2 + 2k + 1) - k^2 = 2k + 1 = x.$$

   Hence x is the difference of two perfect squares, namely, $(k + 1)^2$ and k^2. □

4. **Proposition.** If ten balls are chosen from a box containing only red balls, yellow balls, and green balls, then at least four of the chosen balls are of the same color.

   Proof. Suppose that ten balls are chosen from the box and that no four of the chosen balls are of the same color. Then the chosen balls must include no more than three balls of any one color. Since there are only three colors, the number of chosen balls can be no greater than $3 \times 3 = 9$. But ten balls were chosen. Hence at least four of the chosen balls are of the same color. □