


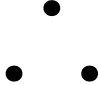
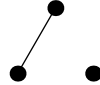
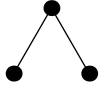
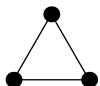
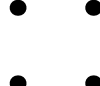
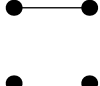
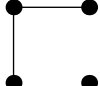
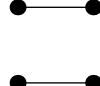
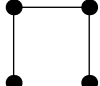
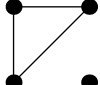
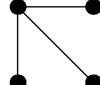
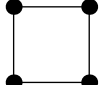
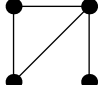
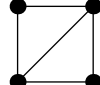
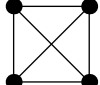


CSCE 235: Introduction to Discrete Structures
Homework assignment 7 (solutions)
Assigned Tuesday, April 10, 2007
Due Friday, April 20, 2007

Problem 1. (18 points) Make a list of drawings of all the simple graphs with at most four vertices. The list should not contain any isomorphic pairs of graphs. (You should get a list of 18 graphs.)

Solution.

| | | |
|---|---|--|
| 1.  | 2.  | 3.  |
| 4.  | 5.  | 6.  |
| 7.  | 8.  | 9.  |
| 10.  | 11.  | 12.  |
| 13.  | 14.  | 15.  |
| 16.  | 17.  | 18.  |

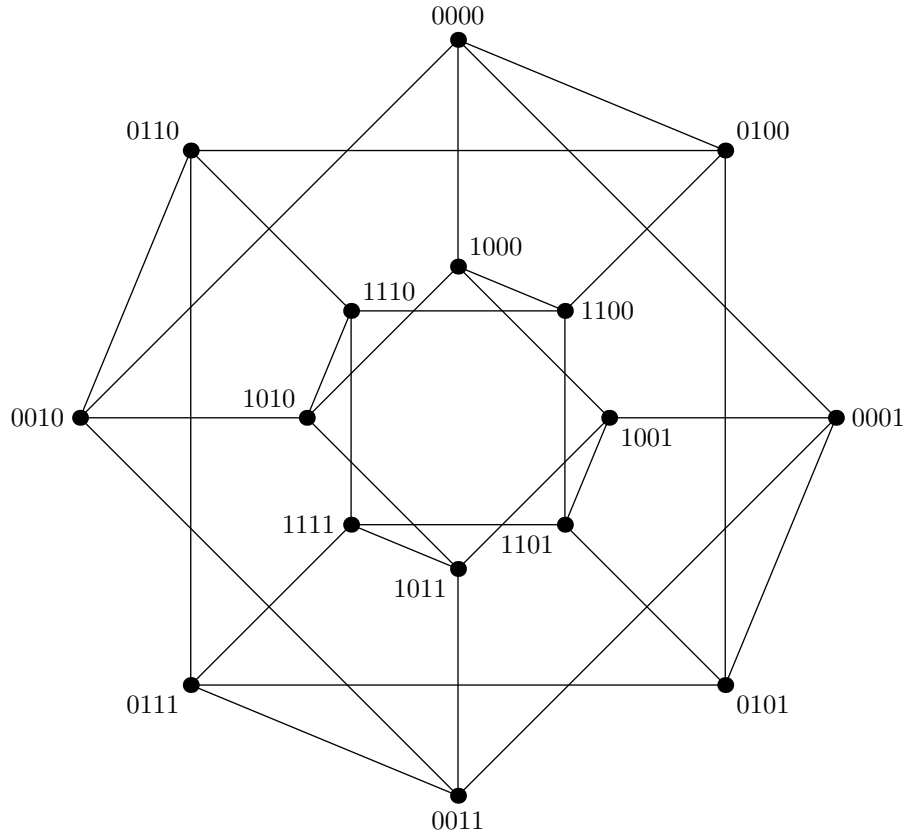
Problem 2. (8 points) Draw a graph with exactly five vertices, each of degree 5, or explain why such a graph does not exist.

Solution. No such graph exists, because by the handshaking theorem (i.e., the degree-sum theorem, page 599), the number of vertices in a graph which have odd degree must be even.

Note that it is *not* a rigorous argument to begin with the complete graph on five vertices, say that each vertex has degree 4, and argue that there is no way to add more edges to bring all the degrees up to 5. For one thing, this argument makes the assumption that the only way it would be possible to make a graph with exactly five vertices each of degree 5 is to start with K_5 , but there is no justification for this assertion. Conceivably, a graph with exactly five vertices each of degree 5 may not have K_5 as a subgraph. The handshaking theorem is required for a rigorous justification for this problem.

Problem 3. (10 points) Draw Q_4 . Is this graph bipartite?

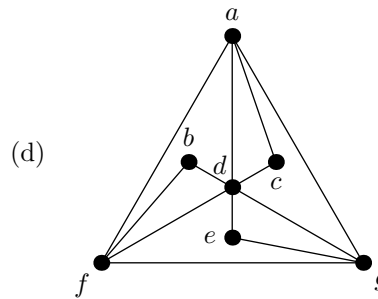
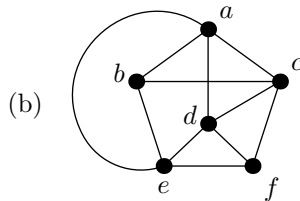
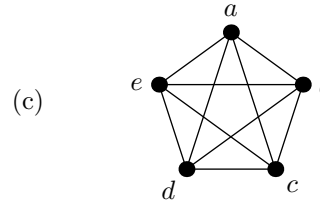
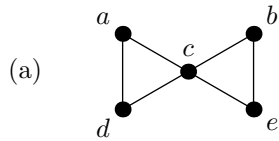
Solution. Q_4 is the graph whose vertices are labeled with all possible bit strings of length 4, in which two vertices are connected by an edge if and only if their labels differ in exactly one position. There are many ways to draw this graph. One way is shown below.



Note that in this drawing, each of the positions of the bit string has a “meaning.” The first bit is 0 for those vertices in the outer ring and 1 for those vertices in the inner ring. The second bit is 0 for those vertices at the four main compass points (north, south, east, and west), and 1 for the vertices in between (northeast, southeast, northwest, and southwest). The third bit is 0 for the vertices to the right of a line that goes through the center of the drawing and through the compass points NNW and SSE, and 1 for the vertices to the left of this line. Finally, the fourth bit is 0 for the vertices above a line that passes through the center of the drawing and through the compass points ENE and WSW, and 1 for the vertices below this line. The symmetry of the drawing above is due in part to this arrangement of the vertices, so that each bit has a meaningful interpretation.

This graph is bipartite, as are all n -cubes. To see this, let A be the set of vertices in Q_4 whose labels contain an even number of 1s, and let B be the set of vertices in Q_4 whose labels contain an odd number of 1s. Clearly every vertex in Q_4 is in either A or B , and no vertex is in both A and B , so A and B form a partition of the vertex set of Q_4 . Since every edge in the graph connects two vertices that differ in exactly one position, every edge connects a vertex with an even number of 1s to a vertex with an odd number of 1s. In other words, there is no edge that connects a vertex in A to another vertex in A , and there is no edge that connects a vertex in B to another vertex in B . Thus the pair (A, B) is a bipartition of the vertex set of Q_4 , and hence Q_4 is bipartite.

Problem 4. (24 points) For each of the graphs below, determine whether it has an Euler circuit, and construct such a circuit if one exists. If the graph has no Euler circuit, explain why. Also determine whether the graph has a Hamiltonian circuit, and either construct such a circuit if one exists or justify your claim that there is no Hamiltonian circuit.



Solution.

(a) Since the degree of every vertex is even, this graph has an Euler circuit; one such circuit is a, c, e, b, c, d, a . However, it does not have a Hamiltonian circuit. Such a circuit would need to include a at some point, then later travel to b , and eventually come back to a . This requires visiting c twice.

(b) This graph has no Euler circuit, since the vertex b has degree 3, which is odd. It does have a Hamiltonian circuit, for example, a, e, d, f, c, b, a .

(c) Since the degree of every vertex is even, this graph has an Euler circuit, for example, $a, b, c, d, e, a, c, e, b, d, a$. It also has a Hamiltonian circuit, for example, a, b, c, d, e, a .

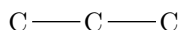
(d) Again, since the degree of every vertex is even, this graph has an Euler circuit, for example, $a, c, d, a, g, e, d, g, f, b, d, f, a$. However, it does not have a Hamiltonian circuit. In order for a Hamiltonian circuit in this graph to reach the vertex b , for example, it would need to pass through d either immediately before b or immediately after. The same goes for the vertices c and e . But a Hamiltonian circuit can visit d only once, so it cannot visit all three of b, c , and e .

Problem 5. (12 points) In graph models of saturated hydrocarbons, each carbon atom is represented by a vertex of degree 4, and each hydrogen atom is represented by a vertex of degree 1. (See Example 5 on page 688 of the textbook.) The nonisomorphic trees with n vertices of degree 4 and $2n + 2$ vertices of degree 1 represent the different isomers of C_nH_{2n+2} . How many isomers are there of

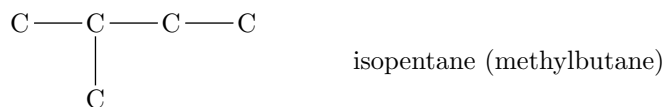
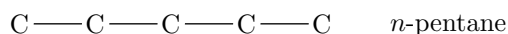
- (a) propane (C_3H_8)?
- (b) pentane (C_5H_{12})?
- (c) hexane (C_6H_{14})?

Solution. It turns out that we can ignore the hydrogen atoms when we draw the molecules; the only thing that matters is the arrangement of the carbon atoms.

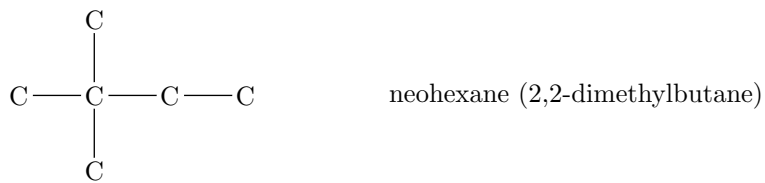
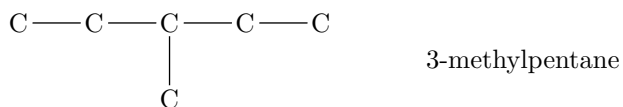
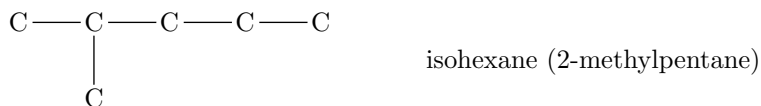
- (a) Propane, C_3H_8 , has just one isomer, shown below.



- (b) Pentane, C_5H_{12} , has three isomers, shown below.

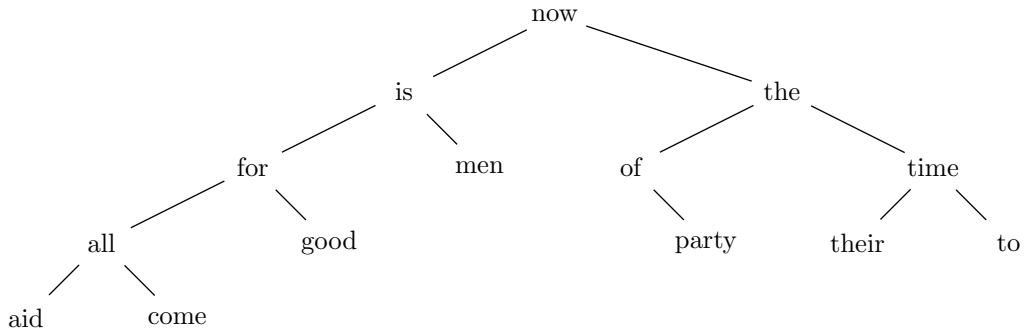


- (c) Hexane, C_6H_{14} , has five isomers, shown below.

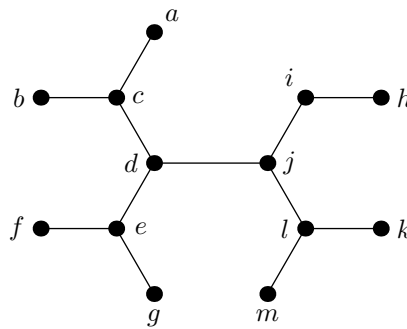


Problem 6. (10 points) Build a binary search tree for the words in the sentence “now is the time for all good men to come to the aid of their party”, inserting the words into the tree in the order they appear in the sentence. Do not insert duplicate words more than once.

Solution.

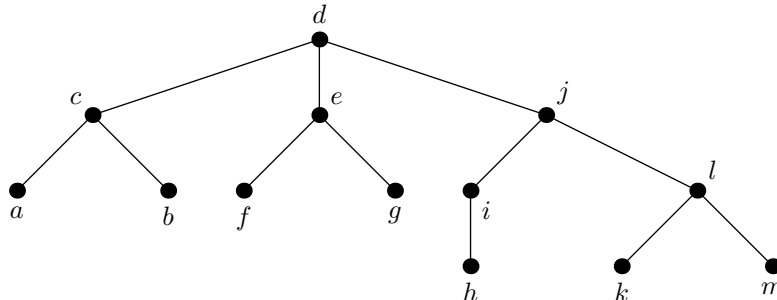


Problem 7. (21 points) Consider the following tree T .



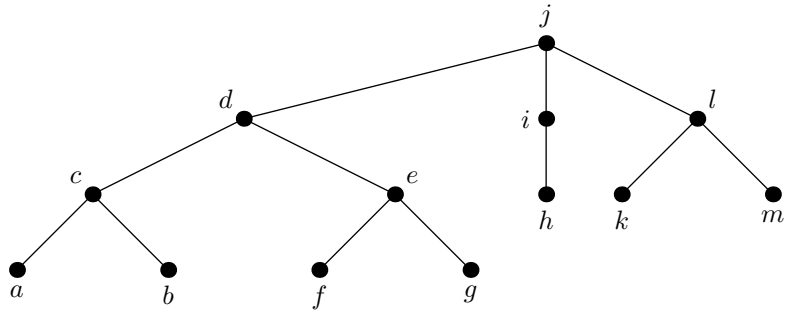
Form an ordered rooted tree T' from T by choosing d as the root and ordering the children of each internal vertex alphabetically. Draw T' . In what order are the vertices encountered in the preorder traversal of T' ? in the inorder traversal of T' ? in the postorder traversal of T' ? Answer the same questions if j is chosen as the root, and if f is chosen as the root.

Solution. We begin by choosing d to be the root. The resulting ordered rooted tree T' is shown below.



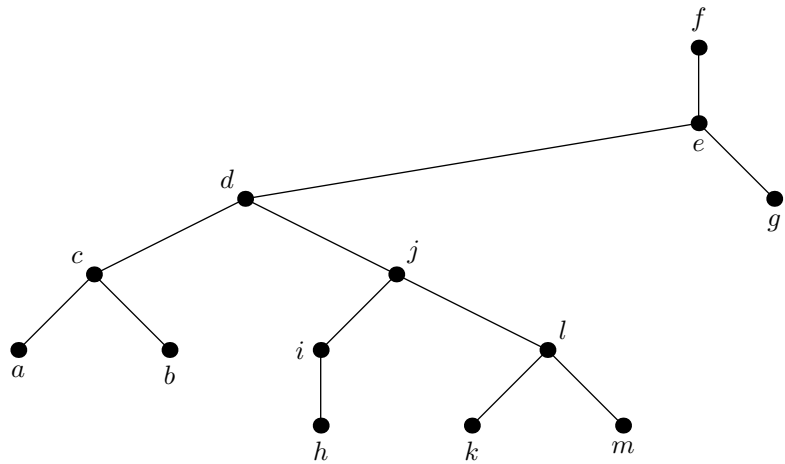
The preorder traversal will visit the vertices of T' in the order $d, c, a, b, e, f, g, j, i, h, l, k, m$. The inorder traversal will visit the vertices in the order $a, c, b, d, f, e, g, h, i, j, k, l, m$. The postorder traversal will visit the vertices in the order $a, b, c, f, g, e, h, i, k, m, l, j, d$.

We now choose j to be the root, and we obtain the ordered rooted tree below.



The preorder traversal of this tree will visit the vertices in the order $j, d, c, a, b, e, f, g, i, h, l, k, m$. The inorder traversal will visit the vertices in the order $a, c, b, d, f, e, g, j, h, i, k, l, m$. The postorder traversal will visit the vertices in the order $a, b, c, f, g, e, d, h, i, k, m, l, j$.

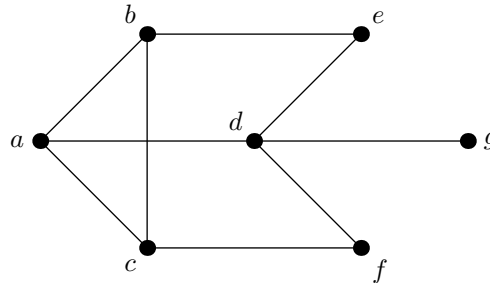
If we choose f as the root, we get the following ordered rooted tree.



The preorder traversal of this tree will visit the vertices in the order $f, e, d, c, a, b, j, i, h, l, k, m, g$. The inorder traversal will visit the vertices in the order $a, c, b, d, h, i, j, k, l, m, e, g, f$. The postorder traversal will visit the vertices in the order $a, b, c, h, i, k, m, l, j, d, g, e, f$.

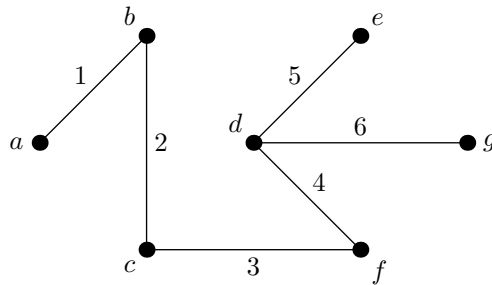
Problem 8. (12 points) Find a spanning tree for the graph shown below by beginning at a and using

- (a) depth-first search.
- (b) breadth-first search.



Solution.

(a) One possible answer is shown below. The numbers on the edges do *not* indicate weights; rather, they indicate the order in which the edges are added to the spanning tree as it is being built.



(b) One possible answer is shown below. Again, the numbers on the edges do not indicate weights; rather, they indicate the order in which the edges are added to the spanning tree as it is being built. With breadth-first search, it is often helpful to imagine whole sets of edges being added at once, which is why three edges are labeled with 1 and the other three are labeled with 2.

