CSCE 235: Introduction to Discrete Structures
Homework assignment 6 (92 points)
Assigned Monday, April 2, 2007
Due Monday, April 9, 2007

Problem 1. (12 points) Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2, \ldots$.

(a) Find a_0, a_1, a_2, a_3, and a_4.
(b) Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$.
(c) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$.

Problem 2. (16 points) Is the sequence $\{a_n\}$ a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ if

(a) $a_n = 0$?
(b) $a_n = 1$?
(c) $a_n = 2^n$?
(d) $a_n = 4^n$?
(e) $a_n = n4^n$?
(f) $a_n = 2 \cdot 4^n + 3n4^n$?
(g) $a_n = (-4)^n$?
(h) $a_n = n^2 4^n$?

Problem 3. (12 points) Assume that the population of the world in 2007 is 6.6 billion and that the population will grow at the constant rate of 1.14% each year.

(a) Set up a recurrence relation for the population of the world n years after 2007.
(b) Find an explicit formula for the population of the world n years after 2007.
(c) What will the population of the world be in 2027?
(d) When will the world population reach 10 billion?

Problem 4. (12 points) A vending machine dispensing books of stamps accepts only dollar coins, \$1 bills, and \$5 bills.

(a) Find a recurrence relation for the number of ways to deposit n dollars into the machine, where the order in which the coins and bills are deposited matters.
(b) What are the initial conditions?
(c) How many ways are there to deposit \$10 for a book of stamps?

Problem 5. (10 points) Find the solution to the recurrence relation

$$a_n = 7a_{n-1} - 10a_{n-2} \qquad \text{for } n \geq 2,$$

with initial conditions $a_0 = 2$ and $a_1 = 1$.

Problem 6. (10 points) Find the solution to the recurrence relation

$$a_n = -6a_{n-1} - 9a_{n-2} \qquad \text{for } n \geq 2,$$

with initial conditions $a_0 = 3$ and $a_1 = -3$.

Problem 7. (10 points) Find the solution to the recurrence relation

$$a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3} \qquad \text{for } n \geq 3,$$

with initial conditions $a_0 = 7$, $a_1 = -4$, and $a_2 = 8$.

Hint: You will need to use Theorem 3 on page 465 of the textbook.

Problem 8. (10 points) The Lucas numbers satisfy the recurrence relation

$$L_n = L_{n-1} + L_{n-2} \qquad \text{for } n \geq 2,$$

with the initial conditions $L_0 = 2$ and $L_1 = 1$. Find an explicit formula for the Lucas numbers.