

CSCE 235: Introduction to Discrete Structures

Homework assignment 5 (111 points)

Assigned Tuesday, March 20, 2007

Due Monday, March 26, 2007

Justify your answers to the following questions carefully.

Problem 1. (10 points) How many strings consist of four upper-case letters and contain the letter Q?

Problem 2. (10 points) A computer program generates integers at random. How many integers must be generated in order to guarantee that at least eight of the integers have the same remainder when divided by 71?

Problem 3. (10 points) Show that if there are 100 million wage earners in the United States who earn less than a million dollars, then there are two who earned exactly the same amount of money, to the penny, last year.

Problem 4. (10 points) A distributed computing project consists of a server that distributes work units to several computers connected to a network. Suppose the server has five different work units to distribute and there are twelve available computers. In how many ways can the work units be distributed if no computer can process more than one work unit?

Problem 5. (10 points) Norbert and Helga are walking on the beach, collecting seashells. They come across a pile of ten seashells, all beautiful and all different. They agree to divide the pile evenly between them. In how many ways can they do this?

Problem 6. (15 points) The parliament of the small country of Brualdia resolves to form a new committee. The committee will be made up of seven members, chosen from the 53 members of parliament. In how many ways can the committee be formed? Suppose that the oldest member of parliament is guaranteed a position on the committee; then how many ways can the committee be formed?

Problem 7. (20 points) Let n be a nonnegative integer. Show that

$$2^n \sum_{k=0}^n \binom{n}{k} = \sum_{k=0}^n 3^k \binom{n}{k}.$$

Hint: One way to do this is to show that the left-hand side and the right-hand side are both equal to 4^n .

Problem 8. (10 points) In how many ways can thirty identical trinkets be distributed among six people? It's fine if some people don't get any trinkets, and only the number of trinkets each person gets matters. Giving Ralph all thirty trinkets is different from giving Susan all thirty trinkets, but giving Ralph the first fifteen trinkets and Susan the last fifteen is the same as giving Susan the first fifteen and Ralph the last fifteen, because all the trinkets are identical.

Problem 9. (16 points) How many different strings can be formed by rearranging the letters in each of the following words?

- (a) ORANGE
- (b) SASSAFRAS
- (c) UNCOPYRIGHTABLE
- (d) EXTRACURRICULAR