CSCE 235: Introduction to Discrete Structures
Homework assignment 4 (120 points)
Assigned Wednesday, February 21, 2007
Due Wednesday, February 28, 2007

Problem 1. (16 points) For each of the following functions, give the domain, codomain, and range of the function; specify whether or not it is one-to-one, whether or not it is onto, whether or not it is bijective, and whether or not it is invertible; and if it is invertible, give its inverse.

(a) $f : \mathbf{Q} \to \mathbf{Q}$ given by $f(x) = \frac{1}{2}x^2 - 4$.
(b) $g : \mathbf{R} \to \mathbf{R}$ given by $g(x) = x^5$.
(c) $h : \mathbf{R} \to \mathbf{Z}$ given by $h(x) = \lfloor x \rfloor$.
(d) $\phi : \mathbf{N} \to \mathbf{N} \times \mathbf{N}$ given by $\phi(n) = (n, n)$.

Problem 2. (8 points) Let $f : \mathbf{R} \to \mathbf{R}$ be given by $f(x) = x^2 + 1$ and let $g : \mathbf{R} \to \mathbf{R}$ be given by $g(x) = 3x + 2$.

(a) Find $f \circ g$.
(b) What is the value of $(f \circ g)(1)$? of $(f \circ g)(-1)$?
(c) Find $g \circ f$.
(d) What is the value of $(g \circ f)(1)$? of $(g \circ f)(-1)$?

Problem 3. (10 points) Find these values.

(a) $\lfloor 1.02 \rfloor$
(b) $\lceil 1.02 \rceil$
(c) $\lfloor -3.8 \rfloor$
(d) $\lceil -3.8 \rceil$
(e) $8!$
(f) $n!/(n-4)!$, where n is an integer greater than 4

Problem 4. (20 points) Let $P(n)$ be the statement that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for the positive integer n. The parts of this problem outline a proof using mathematical induction to show that this formula is true for all positive integers n.

(a) What is the statement $P(1)$?
(b) Show that $P(1)$ is true, completing the basis step of the proof.
(c) What is the inductive hypothesis?
(d) What do you need to prove in the inductive step?
(e) Complete the inductive step.
(f) Explain why these steps show that this formula is true whenever n is a positive integer.

Problem 5. (20 points) Prove that for every positive integer n,

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

Problem 6. (20 points) Let $Q(n)$ be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. The parts of this problem outline a strong induction proof that $Q(n)$ is true for $n \geq 8$.

(a) Show that the statements $Q(8)$, $Q(9)$, and $Q(10)$ are true, completing the basis step of the proof.
(b) What is the inductive hypothesis of the proof?
(c) What do you need to prove in the inductive step?
(d) Complete the inductive step.
(e) Explain why these steps show that this statement is true whenever $n \geq 8$.

Problem 7. (8 points) Find $f(1)$, $f(2)$, $f(3)$, and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$ and for $n = 1, 2, 3, \ldots$,

(a) $f(n) = f(n-1) + 2$.
(b) $f(n) = \big(f(n-1) + 1\big)^2$.

Problem 8. (8 points) Find $f(2)$, $f(3)$, $f(4)$, and $f(5)$ if $f(n)$ is defined recursively by $f(0) = -1$, $f(1) = 2$, and for $n = 2, 3, 4, \ldots$,

(a) $f(n) = f(n-1) + 3f(n-2)$.
(b) $f(n) = f(n-2)/f(n-1)$.