

CSCE 235: Introduction to Discrete Structures

Homework assignment 3 (87 points)

Assigned Wednesday, February 7, 2007

Due Monday, February 12, 2007

Problem 1. (15 points) Prove that for any integer n there does not exist an integer k such that $n^2 = 3k + 2$.

Hint: Use a proof by cases. If any integer is divided by 3, the remainder is either 0, 1, or 2. Therefore every integer can be written as $3k$, $3k + 1$, or $3k + 2$ for some integer k . Consider these three cases separately. For example, if n leaves a remainder of 1 when divided by 3, so that $n = 3k + 1$ for some k , then what is the remainder when n^2 is divided by 3?

Problem 2. (6 points) Let $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{3\}$, and $D = \{1, 2, 3\}$.

- (a) (2 points) Use set builder notation to describe D .
- (b) (4 points) Which of these sets are subsets of which other of these sets? In other words, list all true statements of the form $X \subseteq Y$, where X and Y are to be replaced with A , B , C , or D .

Problem 3. (9 points) Determine whether each of these statements is true or false.

- (a) $0 \in \emptyset$
- (b) $\emptyset \in \{0\}$
- (c) $\{0\} \subseteq \emptyset$
- (d) $\emptyset \subseteq \{0\}$
- (e) $\{0\} \in \{0\}$
- (f) $\{0\} \subseteq \{0\}$
- (g) $\{\emptyset\} \subseteq \{\emptyset\}$
- (h) $\emptyset \in \{\emptyset\}$
- (i) $\emptyset \subseteq \{\emptyset\}$

Problem 4. (5 points) Let A and B be finite sets. Let m be the cardinality of A and let n be the cardinality of B . Find the cardinality of the following sets.

- (a) $P(A)$
- (b) $P(B)$
- (c) $A \times B$
- (d) $P(A \times B)$
- (e) $P(A) \times P(B)$

Problem 5. (8 points) Let

$$\begin{aligned}A &= \{1, 2, 3, 4, 5\}, \\B &= \{x \mid x \text{ is even}\}, \\C &= \{x \mid x \text{ is a multiple of 3}\}, \text{ and} \\D &= \{x \mid x \text{ is prime}\},\end{aligned}$$

where the universal set is the set of all positive integers less than 20. [Note: The number 1 is not prime.]

- (a) What is $A \cup C$?
- (b) What is $B \cap C$?
- (c) What is $A - D$?
- (d) What is $D - A$?
- (e) What is \overline{A} ?
- (f) What is $B \cap D$?
- (g) What is $(B \cap C) \cap D$?
- (h) What is $(B \cup D) \cap \overline{A}$?

Problem 6. (8 points) List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$ where $(a, b) \in R$ if and only if

- (a) $a = b$.
- (b) $a + b = 4$.
- (c) $a > b$.
- (d) a divides b .

Problem 7. (16 points) For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is transitive, and whether it is an equivalence relation.

- (a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- (b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- (c) $\{(1, 2), (2, 3), (3, 4)\}$
- (d) $\{(1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 4)\}$

Problem 8. (10 points) Which of these relations on \mathbf{Z} are equivalence relations? Determine the properties of an equivalence relation that the others lack.

- (a) $\{(x, y) \mid x > y\}$
- (b) $\{(x, y) \mid x = y\}$
- (c) $\{(x, y) \mid |x - y| \leq 1\}$
- (d) $\{(x, y) \mid x \text{ divides } y\}$
- (e) $\{(x, y) \mid |x| = |y|\}$

Problem 9. (10 points) Find a partition of the positive integers such that no two prime numbers are in the same subset and every subset contains a prime number.