CSCE 235: Introduction to Discrete Structures
Homework assignment 1 (70 points)
Assigned Wednesday, January 17, 2007
Due 12:30 p.m., Wednesday, January 24, 2007
(Homework five minutes late will *not* be accepted.)

Problem 1. (4 points) Consider the following propositions.

$$\begin{array}{cccc} p \to q, & \neg p \to \neg q, & q \to p, & \neg q \to \neg p, \\ p \wedge q, & p \wedge \neg q, & \neg p \wedge q, & \neg p \wedge \neg q, \\ p \vee q, & p \vee \neg q, & \neg p \vee q, & \neg p \vee \neg q. \end{array}$$

(a) Which proposition is the converse of $p \to q$?
(b) Which proposition is the contrapositive of $p \to q$?
(c) Which proposition is the inverse of $p \to q$?
(c) Which propositions are logically equivalent to $p \to q$?

Problem 2. (4 points) Suppose that the truth value of $\neg p \to \neg q$ is known to be false. Give the truth values for

(a) $p \wedge q$,
(b) $p \vee q$,
(c) $p \to q$,
(d) $q \to p$.

Problem 3. (8 points) Show that each of these implications is a tautology by using truth tables.

(a) $[p \wedge (p \to q)] \to q$
(b) $[(p \to q) \wedge \neg q] \to \neg p$
(c) $(p \to q) \to [(q \to r) \to (p \to r)]$
(d) $p \to [q \to (p \wedge q)]$

Problem 4. (10 points) Prove or disprove the following. (Hint: Only one line of the truth table is needed to show that a proposition is *not* a tautology.)

(a) $\neg(p \vee q) \iff (\neg p \wedge \neg q)$
(b) $[(p \to q) \wedge \neg p] \implies \neg q$
(c) $[(p \wedge q) \to r] \iff [p \to (q \to r)]$
(d) $(p \to q) \iff (\neg p \to \neg q)$
(e) $(p \vee q) \wedge q \iff \neg(p \wedge q)$

Problem 5. (4 points) Recall that the "exclusive or" connective is denoted by the symbol $\oplus$ (see page 5 of the textbook).

(a) Show that $(p \oplus q) \iff [(p \vee q) \wedge \neg(p \wedge q)]$.
(b) Show that $(p \oplus q) \iff \neg(p \leftrightarrow q)$.

Problem 6. (8 points) Every compound proposition can be written using only the connectives $\neg$ and $\vee$. This fact follows from the equivalences $(p \rightarrow q) \iff (\neg p \vee q)$, $(p \wedge q) \iff \neg(\neg p \vee \neg q)$, and $(p \leftrightarrow q) \iff [(p \rightarrow q) \wedge (q \rightarrow p)]$. Find propositions logically equivalent to the following, using only the connectives $\neg$ and $\vee$.

(a) $p \leftrightarrow q$
(b) $(p \wedge q) \rightarrow (\neg q \wedge r)$
(c) $(p \rightarrow q) \wedge (q \vee r)$
(d) $p \oplus q$ [Hint: See Problem 5(b).]

Problem 7. (12 points) The *Sheffer stroke* is a logical connective written | and defined by the following truth table.

p	q	$p \mid q$
T	T	F
T	F	T
F	T	T
F	F	T

This connective is interesting because Henry M. Sheffer (for whom it is named) proved in 1913 that all compound propositions can be written using only this single connective. You may prove logically or use a truth table (or a combination of both) for the following.

(a) Show that $\neg p \iff p \mid p$.
(b) Show that $(p \vee q) \iff (p \mid p) \mid (q \mid q)$.
(c) Find a proposition equivalent to $p \wedge q$ using only the Sheffer stroke.
(d) Find a proposition equivalent to $p \rightarrow q$ using only the Sheffer stroke.
(e) For **three bonus points**, find a proposition equivalent to $p \leftrightarrow q$ using only the Sheffer stroke. What is the minimum number of Sheffer strokes required?

Problem 8. (10 points) Complete the following formal proofs by supplying an explanation for each step.

(a) If $p \rightarrow (q \vee \neg r)$, $\neg(r \rightarrow q)$, and $p \vee s$, then s.

Proof:

1. $p \rightarrow (q \vee \neg r)$
2. $\neg(r \rightarrow q)$
3. $p \vee s$
4. $\neg(\neg r \vee q)$
5. $\neg(q \vee \neg r)$
6. $\neg p$
7. s

(b) If $p \to (q \to r)$, $p \vee \neg s$, and q, then $s \to r$.

Proof:

1. $p \to (q \to r)$
2. $p \vee \neg s$
3. q
4. $\neg s \vee p$
5. $s \to p$
6. $s \to (q \to r)$
7. $(s \wedge q) \to r$
8. $q \to [s \to (q \wedge s)]$
9. $s \to (q \wedge s)$
10. $s \to (s \wedge q)$
11. $s \to r$

Problem 9. (10 points) Complete the following proofs by contradiction by supplying an explanation for each step.

(a) If $p \to (\neg q \wedge r)$, q, and $p \vee s$, then s.

Proof:

1. $p \to (\neg q \wedge r)$
2. q
3. $p \vee s$
4. $\neg s$
5. $s \vee p$
6. $\neg(\neg s) \vee p$
7. $\neg s \to p$
8. p
9. $\neg q \wedge r$
10. $\neg q$
11. $q \wedge (\neg q)$
12. contradiction

(b) If $q \wedge (r \wedge p)$, $t \to v$, and $v \to \neg p$, then $\neg t \wedge r$.

Proof:

1. $q \wedge (r \wedge p)$
2. $t \to v$
3. $v \to \neg p$
4. $\neg(\neg t \wedge r)$
5. $t \vee \neg r$
6. $\neg r \vee t$
7. $r \to t$
8. $r \to v$
9. $r \to \neg p$
10. q
11. $r \wedge p$
12. r
13. p
14. $\neg p$
15. $\neg p \wedge p$
16. contradiction