Multiple Third Order Cyclic Frequencies Based Spectrum Sensing Scheme for CR Networks

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Abstract—**A new spectrum sensing scheme based on the multiple third order cyclic frequencies in cognitive radio networks is proposed in this paper. The binary hypothesis test model and the test statistic are constructed to derive the explicit formula of the detection probability by exploiting the asymptotic optimal estimation theory. The computational complexity of the proposed scheme is nearly as the same order of magnitude as that of energy detection (ED) by comparing the real multiplication operation times of them. At last, numerical results are demonstrated to illustrate that our new scheme has superior performances with lower computational complexity.**

Keywords- spectrum sensing; third order cyclic frequency; detection probability; computational complexity

I. INTRODUCTION

With the rapid development of wireless technologies over the past several years, more and more spectrum resources are needed to satisfy numerous emerging wireless services. The contradiction between the limitation of wireless spectrum and the increasing requirement of wireless communication services becomes more serious for the current spectrum regulatory framework, in which all of the frequency bands are exclusively allocated to specific services and no violation from unlicensed users is allowed. To solve this problem, CR (Cognitive Radio) is envisioned to provide opportunistic spectrum access by extending the underutilized, licensed frequency bands to secondary users (SUs) without causing harmful interference to the primary users (PUs). Spectrum sensing plays an important role in cognitive radio for the SUs need to reliably and swiftly detect the presence of the PUs. Evidently, particular attention should be paid to design of spectrum sensing technique for SUs to detect the unused frequency bands in cognitive radio networks.

The most concerning fields about the PU transmitter detection based non-cooperative spectrum sensing techniques can be divided into three categories, which are ED, cyclostationary feature detection (CFD) and matched filter detection (MFD) [1], [2]. Within these schemes, ED is one of the most convenient techniques to detect the presence of the PU signal if the power of noise is known exactly at the receiver. However, the uncertainty in the power estimation of noise will significantly impact the performance of ED [3]. CFD exploits the built-in periodicity of the PU signal to sense the channel. It differentiates the noise energy from modulated PU signal energy, which results the fact that the noise is a wide-sense stationary signal with no cyclic correlation, while PU signal is cyclostationary with spectral correlation [2]. For MFD, the applicability of it is limited because it requires a priori knowledge of the PU signal during the course of detection [1].

Since the second order cyclic statistics can not provide enough cyclostationary information, it is inadequate to sense the channel reliably for the second order cyclostationary feature detection (SOCFD) in certain situations, e.g. the quadrature amplitude modulation (QAM) television signals [4], [5]. Because of the virtue of the cyclostationarity in strict sense, the third order cyclic statistics can separate the stationary signal and the cyclostationarity signal from other signal completely; restrain the stationary noise and non-stationary Gaussian noise [6]. The third order cyclic statistics can to be used for spectrum sensing in cognitive radio according to the third order cyclostationarity, which is detected by using hypothesis testing. The testing strategy developed the tests to check for the presence of the cyclic frequencies by exhaustively searching over the candidate cyclic frequencies for which the corresponding cyclic statistics are nonzero and statistically significant [5]. The asymptotic optimal strategy can also be introduced to construct the binary hypothesis test model, and to derive the test statistic for having spectrum sensing rather than to detect the presence of the cyclostationarity. A PU signal may have the multiple third order cyclic frequencies (MTOCF) related to the carrier frequency, the symbol rate and corresponding harmonics, the chip rate, guard period, the channel coding scheme and so on. These cyclic frequencies can be used for collaborative sensing without considering the channel condition among the collaborative users. The explicit expression of detection probability may be derived by using the asymptotic optimal estimation theory, while the computational complexity of the proposed sensing scheme can also be analyzed through the actual operation times of multiplication.

II. FUNDAMENTALS OF SPECTRUM SENSING BASED ON THE THIRD ORDER CYCLIC CUMULANT

A. Fundamentals of the Third Order Cyclostationarity

To make the derivation of the detector in the following sections, a brief review of the basic concepts and definitions

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associated with the third order cyclostationary processes are presented in this section.

A zero mean process $x(t)$ is said to be the third order cyclostationary if the third order time varying moment of it is a periodic function with parameter T,

$$m_{3x}(t;\tau_1,\tau_2)=m_{3x}(t+T;\tau_1,\tau_2)\,. \tag{1}$$

For some cycle time $T\neq 0$

$$m_{3x}(t;\tau_1,\tau_2)=E\{x(t)x(t+\tau_1)x(t+\tau_2)\} \tag{2}$$

where, $E\{x(t)x(t+\tau_1)x(t+\tau_2)\}$ is the mathematical expectation of $x(t)x(t+\tau_1)x(t+\tau_2)$. $m_{3x}(t;\tau_1,\tau_2)$ can be represented by a Fourier series because of the periodic character of it.

$$m_{3x}(t;\tau_1,\tau_2)=\sum_{m=-\infty}^{\infty}M_{3x}^{\alpha}(\tau_1,\tau_2)e^{j2\pi\alpha t} \tag{3}$$

$\alpha=m/T$ ($m\in Z$) is the cyclic frequency in equation (3). The sum over α includes all integer multiples of the reciprocal in the unit period T . The Fourier coefficient $M_{3x}^{\alpha}(\tau_1,\tau_2)$, which is called the third order cyclic moment at the cyclic frequency α of $x(t)$, is given by

$$M_{3x}^{\alpha}(\tau_1,\tau_2)=\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}m_{3x}(t;\tau_1,\tau_2)e^{-j\alpha t}=\left\langle m_{3x}(t;\tau_1,\tau_2)e^{-j\alpha t}\right\rangle_t\,. \tag{4}$$

It is clearly that the third order cyclic moment is not identically zero for all nonzero α if and only if the third order cyclic moment contains an additive periodic component. This will not be true unless $x(t)$ is the third cyclic cyclostationary. The countable set $A_3^m=\{\alpha : M_{3x}^{\alpha}(\tau_1,\tau_2)\neq 0,\ 0\le\alpha<2\pi\}$ is referred as the set of cyclic frequencies, so equation (3) can be written as

$$m_{3x}(t;\tau_1,\tau_2)=\sum_{\alpha\in A_3^m}M_{3x}^{\alpha}(\tau_1,\tau_2)e^{j2\pi\alpha t} \tag{5}$$

The time varying mean of $x(t)$ can be calculated by

$$M_x(t)=E\{x(t)\}=\sum_{\alpha\in A_1}M_x^{\alpha}e^{j2\pi\alpha t}\,. \tag{6}$$

where $A_1=\{\alpha : M_x^{\alpha}\neq 0,\ 0\le\alpha<2\pi\}$ is the cyclic frequencies set and M_x^{α} is the cyclic mean. Once the cyclic frequencies set is known, $M_x(t)$ can be consistently estimated as

$$\hat{M}_x(t)=\sum_{\alpha\in A_1}\hat{M}_x^{\alpha}e^{j2\pi\alpha t}\,. \tag{7}$$

The Fourier coefficient $\hat{M}_x^{\alpha}$ can be expressed by

$$\hat{M}_x^{\alpha}=\frac{1}{T}\sum_{t=0}^{T-1}x(t)e^{-j2\pi\alpha t}\,. \tag{8}$$

Therefore, the zero mean assumption about the cyclostationary process $x(t)$ does not sacrifice any generality, for if it is not zero mean, one can estimate and subtract the mean. The third order cyclic moment is identical to the third order cyclic cumulant (TOCC) for the zero mean process, so does their cyclic frequencies set [7], i.e.

$$M_{3x}^{\alpha}(\tau_1,\tau_2)=C_{3x}^{\alpha}(\tau_1,\tau_2) \tag{9}$$

$$A_3^m=A_3^c=\{\alpha : M_{3x}^{\alpha}(\tau_1,\tau_2)\neq 0,\ 0\le\alpha<2\pi\}\,. \tag{10}$$

By analogy with the terminology for the conventional third cumulant (which is equation (4) with $\alpha=0$), the Fourier transform of the TOCC,

$$S_{3x}^{\alpha}(w_1,w_2)=\sum_{\tau_1=-\infty}^{\infty}\sum_{\tau_2=-\infty}^{\infty}C_{3x}^{\alpha}(\tau_1,\tau_2)e^{-j(\tau_1w_1+\tau_2w_2)}, \tag{11}$$

is called the cyclic bi-spectrum. The signal $x(t)$ is the third order cyclostationary if one of the following terms is satisfied:

- $M_{3x}^{\alpha}(\tau_1,\tau_2)\neq 0$ when $\alpha\in A_3^m$ and $\alpha\neq 0$;
- $S_{3x}^{\alpha}(w_1,w_2)\neq 0$ when $\alpha\in A_3^c$ and $\alpha\neq 0$.

The third cyclic moment and the cyclic bi-spectrum of the signal $x(t)$, which comprises K independent signals $x_i(t)$, $x(t)=\sum_{i=1}^{K}x_i(t)$, can be expressed as Ref.[6].

$$M_{3x}^{\alpha}(\tau_1,\tau_2)=\sum_{i=1}^{K}M_{3x_i}^{\alpha}(\tau_1,\tau_2) \tag{12}$$

(1) (1)

$$S_{3x}^{\alpha}(w_1,w_2)=\sum_{i=1}^{K}S_{3x_i}^{\alpha}(w_1,w_2) \tag{13}$$

If α is the third order cyclic frequency of the unique signal $x_i(t)$ which is contained in $x(t)$, the expressions (12) and (13) can be written as

$$M_{3x}^{\alpha}(\tau_1,\tau_2)=M_{3x_i}^{\alpha}(\tau_1,\tau_2) \tag{14}$$

$$S_{3x}^{\alpha}(w_1,w_2)=S_{3x_i}^{\alpha}(w_1,w_2)\,. \tag{15}$$

Hence, for a given cyclic frequency of the PU signal $x_i(t)$, which is defined as α_i , by determining that if α_i is the cyclic frequency of the signals received by the SU containing the PU signal, interferences and noises, we can estimate whether or not the PU signal is presented. The channel is considered to be busy if α_i is the cyclic frequency of the signals received by the SU; otherwise, the channel is idle.

B. Spectrum Sensing Based on the TOCC

The basic hypothesis test model for spectrum sensing based on non-cooperative detection can be defined as follows:

$$\begin{cases} H_0 : x(t) = \sum_{i=1}^{K} d_i(t) + n(t) \\ H_1 : x(t) = hs(t) + \sum_{i=1}^{K} d_i(t) + n(t) \end{cases} \tag{16}$$

where

- $x(t)$ the signal received by the SU;
- h the channel gain;
- $s(t)$ the transmitted signal of the PU;
- $n(t)$ the additive white Gaussian noise (AWGN);
- $d_i(t)$ the interference from the other user in the observations.

H_0 is a null hypothesis, which states that there is no PU signal in a certain spectrum band; while H_1 is an alternative hypothesis, which indicates that there exist PU signal. The interference signal and the PU signal have different cyclic frequency since it is impossible that their modulation mode, carrier frequency, symbol rate and channel coding scheme are completely identical. The channel is deemed to be occupied if the nonzero cyclic frequency α, which is the third order cyclic frequency of the PU signal, is as that of the signal received by the SU according to the equation (12)~(15).

Constructing test statistic and formulating corresponding hypothesis testing problem is the main method to detect the presence of cyclostationary feature signals in the practical application of the spectrum sensing for cognitive radio networks. The calculation of the third cyclic cumulant in (4) needs to process infinite data, but it is unrealistic. So for a given the third order cyclic frequency α, we consider the following consistent estimator of $C_{3x}^{\alpha}(\tau_1, \tau_2)$:

$$\begin{aligned} \hat{C}_{3x}^{\alpha}(\tau_1, \tau_2) &= \frac{1}{N} \sum_{t=0}^{N-1} x(t)x(t+\tau_1)x(t+\tau_2)e^{-j\alpha t} \\ &= C_{3x}^{\alpha}(\tau_1, \tau_2) + \varepsilon_{3x}^{\alpha}(\tau_1, \tau_2) \end{aligned} \tag{17}$$

where $\varepsilon_{3x}^{\alpha}(\tau_1, \tau_2)$ represents the estimation error which vanishes asymptotically as $N \to \infty$, N is the number of the samples and $C_{3x}^{\alpha}(\tau_1, \tau_2)$ is the true value of the TOCC. Due to the presence of the estimation error $\varepsilon_{3x}^{\alpha}(\tau_1, \tau_2)$, the estimator $\hat{C}_{3x}^{\alpha}(\tau_1, \tau_2)$ is seldom exactly zero in practice even if α is not the third order cyclic frequency. This problem can be statistically solved by formulating the decision-making problem in a generalized hypotheses testing framework.

Let τ_1, τ_2 be the fixed lags and α be the third order cyclic frequency of the PU signal, the TOCC consistent estimator can be calculated by (17). Now we construct a vector

$$\hat{c}_{3x}^{\alpha}(\tau_1, \tau_2) = \left[\mathrm{Re}\{\hat{C}_{3x}^{\alpha}(\tau_1, \tau_2)\}, \ \mathrm{Im}\{\hat{C}_{3x}^{\alpha}(\tau_1, \tau_2)\} \right] \tag{18}$$

and a matrix

$$\sum\nolimits_{3c}^{\alpha} = \begin{bmatrix} \mathrm{Re}\{P+Q\} & \mathrm{Im}\{P-Q\} \\ \mathrm{Im}\{P+Q\} & \mathrm{Re}\{Q-P\} \end{bmatrix}. \tag{19}$$

Here, $\hat{c}_{3x}^{\alpha}(\tau_1, \tau_2)$ denotes a row vector containing the real and imaginary parts of the TOCC estimator $\hat{C}_{3x}^{\alpha}(\tau_1, \tau_2)$ at the cyclic frequency α; while the matrix $\sum_{3c}^{\alpha}$ represents the covariance matrix of $\hat{c}_{3x}^{\alpha}(\tau_1, \tau_2)$. P and Q in the covariance matrix can be computed by the following expressions:

$$P = \frac{1}{2NL} \sum_{s=-(L-1)/2}^{(L-1)/2} W(s)F(\alpha - \frac{2\pi s}{N})F(\alpha + \frac{2\pi s}{N}) \tag{20}$$

$$Q = \frac{1}{2NL} \sum_{s=-(L-1)/2}^{(L-1)/2} W(s)F^*(\alpha + \frac{2\pi s}{N})F(\alpha + \frac{2\pi s}{N}), \tag{21}$$

where

$$F(w) = \sum_{t=0}^{N-1} x(t)x(t+\tau_1)x(t+\tau_2)e^{-jwt} \tag{22}$$

and $W(s)$ is a spectral window with length L (odd). So the test statistic can be formulated as

$$T_{3c}^{\alpha} = N\hat{c}_{3x}^{\alpha} \sum\nolimits_{3c}^{\alpha-1} \left[\hat{c}_{3x}^{\alpha}\right]^T \tag{23}$$

where $\sum_{3c}^{\alpha-1}$ is the inverse matrix of $\sum_{3c}^{\alpha}$ and $\left[\hat{c}_{3x}^{\alpha}\right]^T$ is the transposed vector of $\hat{c}_{3x}^{\alpha}$. The binary hypothesis test model based on the test statistic in equation (23) can be formulated as:

$$\begin{cases} H_0 : T_{3c}^{\alpha} < \gamma, & \text{PU signal is absent} \\ H_1 : T_{3c}^{\alpha} \geq \gamma, & \text{PU signal is present} \end{cases} \tag{24}$$

in which, γ is the decision threshold.

III. SPECTRUM SENSING SCHEME BASED ON MULTIPLE THIRD ORDER CYCLIC FREQUENCIES

A. Spectrum Sensing Scheme Based on MTOCF

The above-sited hypothesis test has asymptotically constant false alarm rate (CFAR) for testing presence of the third order cyclostationarity at only a given cyclic frequency. It partly ignores the rich information presenting in the signals since a PU signal may have MTOCF related to the carrier frequency, the chip rate, guard period, the channel coding scheme, etc. These cyclic frequencies can be used for collaborative sensing without considering the channel condition among the collaborative users because they are implemented at a single SU. It can be derived from (11) that $C_{3x}^{\alpha}(\tau_1,\tau_2)$ is the Fourier coefficient of the third order cumulant. The Fourier coefficients of a wide-sense stationary random process for different frequencies are asymptotically uncorrelated [8]. The following test statistic is formulated in order to extend the test for the presence of the third order cyclostationarity at any of the concerning cyclic frequencies in set A_3^c simultaneously and retain the CFAR property over a set of tested frequencies:

$$T_{3c} = \sum_{\alpha\in A_3^c} T_{3c}^{\alpha} = N\sum_{\alpha\in A_3^c} \hat{c}_{3x}^{\alpha}\sum\nolimits_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T . \quad (25)$$

The test statistic calculates the sum of the test statistic (equation (23)) over multiple interesting the third order cyclic frequencies in the set A_3^c. Depending on the differences of the signals and the set of adopted cyclic frequencies the test statistic may have different spectrum sensing performances [9].

Now, the hypothesis testing problem based on the test statistic T_{3c} can be formulated as

$$\begin{cases} H_0 : T_{3c} < \gamma', & \text{PU signal is absent} \\ H_1 : T_{3c} \geq \gamma', & \text{PU signal is present} \end{cases} \quad (26)$$

where γ' is the decision threshold.

B. Performance Analysis

Set the decision threshold γ as (24) and γ' as (26), the asymptotic distribution of T_{3c}^{α} and T_{3c} are derived as follows.

According to the Theorem 2 in Ref. [5], the test statistic in equation (23) has the following asymptotic distribution under the null hypothesis H_0

$$T_{3c}^{\alpha} \sim \chi_2^2 \quad (27)$$

and under the alternative hypothesis H_1

$$T_{3c}^{\alpha} \sim \mathcal{N}(N\hat{c}_{3x}^{\alpha}\sum\nolimits_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T,\ 4N\hat{c}_{3x}^{\alpha}\sum\nolimits_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T). \quad (28)$$

Here, we give the Theorem 2 in Ref. [5] in the following.

Theorem 1 (appeared as Theorem 2 in Ref. [5]):

R1) If the $1\times M$ vector $\hat{\theta}$ is an asymptotically Gaussian estimator of θ, based on N data samples, i.e. $\lim_{N\to\infty}\sqrt{N}(\hat{\theta}-\theta)=\mathcal{N}(0,\Sigma_\theta)$ and $\theta\neq 0$, then $\lim_{N\to\infty}\sqrt{N}(\hat{\theta}\hat{\theta}'-\theta\theta')=\mathcal{N}(0,4\theta\,\Sigma_\theta\theta')$, but if $\theta\equiv 0$ and $\Sigma_\theta\equiv I$, then $\lim_{N\to\infty}\sqrt{N}(\hat{\theta}\hat{\theta}'-\theta\theta')=\chi_M^2$. I is an identity matrix, the notation $\mathcal{N}(\mu,\sigma^2)$ denotes a normal probability density function (PDF) with mean μ and variance σ^2, χ_M^2 denotes a central chisquared distribution with M degrees of freedom.

R2) If $\lim_{N\to\infty}\hat{\theta}=\theta$ and $\lim_{N\to\infty}\hat{\psi}\overset{m.s.s}{=}\psi$, an $M\times M$ matrix, then $\lim_{N\to\infty}\hat{\theta}\hat{\psi}\overset{m.s.s}{=}\theta\psi$.

Assuming that the consistent estimation of the TOCC under different cyclic frequencies is independent, the test statistic T_{3c} has the following asymptotic distribution under the null hypothesis H_0

$$T_{3c} \sim \chi_{2N_\alpha}^2 \quad (29)$$

where N_α is the number of adopted cyclic frequencies in set A_3^c and under the alternative hypothesis H_1

$$T_{3c} \sim \mathcal{N}(N\sum_{\alpha\in A_3^c}\hat{c}_{3x}^{\alpha}\sum\nolimits_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T,\ 4N\sum_{\alpha\in A_3^c}\hat{c}_{3x}^{\alpha}\sum\nolimits_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T) . \quad (30)$$

So the false alarm rate and the detection probability of the spectrum sensing scheme based on MTOCF can be obtained by the following formulas:

$$P_{FA} = \Pr\{T_{3c} > \gamma'; H_0\} = Q_{\chi_{2N_\alpha}^2}(\gamma') \quad (31)$$

$$\begin{aligned} P_D &= \Pr\{T_{3c} > \gamma'; H_1\} \\ &= \Pr\left\{ \frac{T_{3c} - N\sum_{\alpha\in A_3^c}\hat{c}_{3x}^{\alpha}\sum_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T}{2\sqrt{N\sum_{\alpha\in A_3^c}\hat{c}_{3x}^{\alpha}\sum_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T}} > \frac{\gamma' - N\sum_{\alpha\in A_3^c}\hat{c}_{3x}^{\alpha}\sum_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T}{2\sqrt{N\sum_{\alpha\in A_3^c}\hat{c}_{3x}^{\alpha}\sum_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T}}; H_1 \right\} \\ &= Q\left(\frac{\gamma' - N\sum_{\alpha\in A_3^c}\hat{c}_{3x}^{\alpha}\sum_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T}{2\sqrt{N\sum_{\alpha\in A_3^c}\hat{c}_{3x}^{\alpha}\sum_{3c}^{\alpha-1}\left[\hat{c}_{3x}^{\alpha}\right]^T}} \right) \end{aligned} \quad (32)$$

where P_{FA} is the false alarm rate, P_D is the detection probability, $Q_{\chi_{2N_\alpha}^2}(\bullet)$ is the right tail probability for a $\chi_{2N_\alpha}^2$ random variable with $2N_\alpha$ degrees of freedom which can be expressed as

$$Q_{\chi^2_{2N_\alpha}}(x)=\int_x^{+\infty}\frac{1}{2^{N_\alpha/2}\Gamma\left(N_\alpha/2\right)}t^{\frac{N_\alpha}{2}-1}\exp(-\frac{1}{2}t)\mathrm{d}t \quad x>0 \tag{33}$$

in which, $\Gamma(\cdot)$ is the gamma function defined as

$$\Gamma(u)=\int_0^{+\infty}t^{u-1}e^{-t}dt \tag{34}$$

and

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{+\infty}\exp(-\frac{1}{2}t^2)dt \tag{35}$$

is the right tail probability of a Gaussian random variable.

$Q_{\chi^2_{2N_\alpha}}(x)$ is a monotonically decreasing function and has an inverse function $Q^{-1}_{\chi^2_{2N_\alpha}}(\bullet)$,so the equation (31) can be written as

$$\gamma'=Q^{-1}_{\chi^2_{2N_\alpha}}(P_{FA})\,. \tag{36}$$

So we can obtain the decision threshold of the test statistic in equation (25) for a given goal false alarm rate P_{FA} from equation (36). The detection probability under a CFAR according to the Neyman-Pearson Theorem in Ref. [10] can be expressed as

$$P_D=Q\left(\frac{Q^{-1}_{\chi^2_{2N_\alpha}}(P_{FA})-N\sum\limits_{\alpha\in A_3^c}\hat{c}^{\alpha}_{3x}\sum{}^{\alpha-1}_{3c}\left[\hat{c}^{\alpha}_{3x}\right]^T}{2\sqrt{N\sum\limits_{\alpha\in A_3^c}\hat{c}^{\alpha}_{3x}\sum{}^{\alpha-1}_{3c}\left[\hat{c}^{\alpha}_{3x}\right]^T}}\right) \tag{37}$$

C. Computational Complexity Analysis

Given the sample number *N*, spectral window length *L* and the threshold γ to reach a CFAR, the SOCFD scheme has to carry out $6NL+4L+3N+10$ times of the real multiplication operation, while the TOCC detection (TOCCD) scheme and the MTOCF detection (MTOCFD) scheme need to carry out $8NL+4L+4N+10$ and $N_\alpha(8NL+4L+4N+10)$ times of those respectively. Considering that the calculation of $F(w)$ in (22) is to calculate the Fourier transform of $x(t)x(t+\tau_1)x(t+\tau_2)$ and the calculation of $\hat{C}^{\alpha}_{3x}(\tau_1,\tau_2)$ in (17) looks like to calculate the Fourier transform of $x(t)x(t+\tau_1)x(t+\tau_2)$, we can firstly calculate the Fourier transform of $x(t)x(t+\tau_1)x(t+\tau_2)$ by using FFT (Fast Fourier Transform) algorithm $X[k]=\mathcal{F}\left(x[n]x[n+\tau_1]x[n+\tau_2]\right)$. Therefore, $\hat{C}^{\alpha}_{3x}(\tau_1,\tau_2)$ and $F(w)$ can be directly obtained from $X[k]$. It can be seen that the spectrum sensing schemes base on the TOCCD and the MTOCFD with implementing the FFT operation firstly demand $4N\log_2^N+4L+2N+10$ and $4N\log_2^N+4LN_\alpha+2N+9N_\alpha+1$ times of the real multiplication operation, respectively. They can both be considered as O(*N*). It approaches the computational complexity of the spectrum sensing scheme based on ED.

IV. NUMERICAL RESULTS AND DISCUSSION

The IEEE 802.22 standard is the first wireless standard based on the cognitive radio. In IEEE 802.22 cognitive radio networks, the television signal is usually chosen as the PU signal [11], and in DVB (Digital Video Broadcasting) Blue-book, QPSK (Quadrature Phase Shift Keying) and the QAM are commonly used television signal modulation schemes [4]. Here a QPSK signal with AWGN is used as the signal received by the SU in the simulations. The QPSK signal is

$$\begin{aligned}s(t)=&\sum_{n=-\infty}^{\infty}a_nq(t-nT_0-t_0)\cos(2\pi f_ct+\phi_0)\\&-\sum_{n=-\infty}^{\infty}b_nq(t-nT_0-t_0)\sin(2\pi f_ct+\phi_0)\end{aligned} \tag{38}$$

where

- a_n the independent and identically distributed random variables that get value 1 or -1 with equal probability;
- b_n the independent and identically distributed random variables that get value 1 or -1 with equal probability;
- T_0 the symbol length;
- $q(t)$ the rectangular pulse of length T_0;
- f_c the carrier frequency;
- ϕ_0 the initial phase of the QPSK signal;
- t_0 the initial time.

The above QPSK signal exhibits the third order cyclostationarity with the cyclic frequencies of $\alpha=p/T_0\pm f_c$ and $\alpha=p/T_0\pm 3f_c$ where $p\in Z$. The number of samples is chosen as *N*=256 and the cyclic spectrum estimations are calculated using a length of *L*=*N*/4-1 Kaiser window with the β parameter 10. All the following sensing performance curves were plotted over 10000 Monte Carlo runs.

Fig. 1 depicts the detection performances of spectrum sensing techniques, which are the sensing schemes based on ED, SOCFD, TOCCD and MTOCFD, as a function of signal to noise ratio (SNR) for a CFAR of 0.01. The 2TOCFD scheme used two the third cyclic frequencies $\alpha=f_c$ and $\alpha=3f_c$, i.e. *M*=2 in the MTOCFD, and the 3TOCFD scheme used three the third order cyclic frequencies $\alpha=f_c$, $\alpha=f_c+1/T_0$ and $\alpha=3f_c$, i.e. *M*=3. Fig. 2 demonstrates the receiver operating characteristic (ROC) curves of the spectrum sensing techniques under SNR= -15dB . The same number of samples was used in the simulations of the spectrum sensing schemes to better illuminate the detection performances of the schemes because

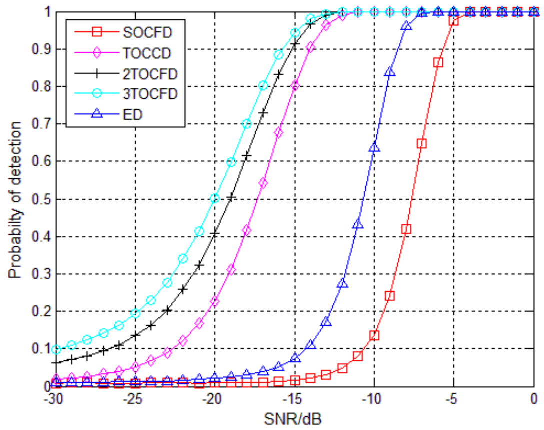

Fig. 1. Probability of detection vs. SNR for $P_{FA} = 0.01$.

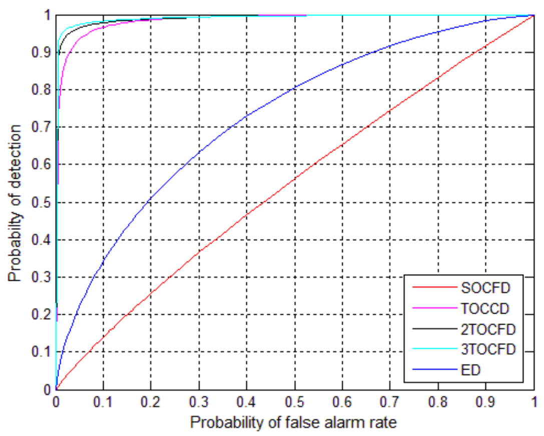

Fig. 2. Probability of detection vs. false alarm rate for SNR=-15dB.

the larger number of samples can improve the detection performance. It can be seen that the performances of the spectrum sensing scheme based on the MTOCFD outperform the performances of the spectrum sensing schemes based on TOCCD, ED and SOCFD in the low SNR regime. Here the energy detector has better performances than those SOCFD because the ideal energy detector with knowing the exact value of the noise power is adopted in our simulations. Furthermore, the spectrum sensing scheme based on the 3TOCFD has better sensing performances than the spectrum sensing scheme based on the 2TOCFD.

The computational complexities of the spectrum sensing schemes with N=256 and L=N/4-1 are shown in Fig. 3. We can conclude that the computational complexity of the spectrum sensing scheme based on the MTOCFD proposed in this paper has not been increased obviously compared to that of the TOCCD and the SOCFD schemes and is nearly as the same order magnitude as that of the ED scheme.

V. CONCLUSION

A novel detection scheme based on the multiple third order cyclic frequencies in cognitive radio systems is proposed in this

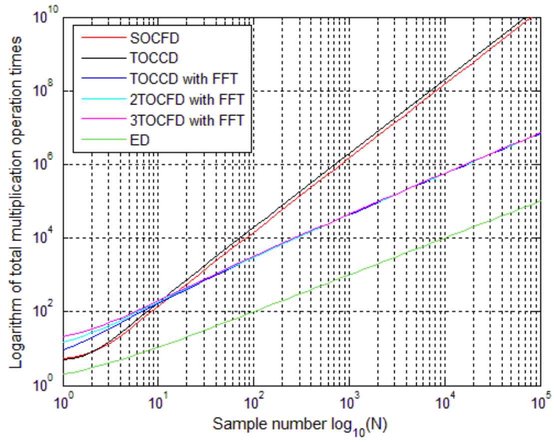

Fig. 3. Computational complexity of the spectrum sensing schemes with N=256 and L=N/4-1.

paper since communication signals often have many periodic statistical properties related to the symbol rate, the coding and modulation schemes. It has the better sensing performances than other traditional non-cooperative spectrum sensing schemes with lower computational complexity which is nearly as the same order magnitude as that of ED scheme. Simulation results show that the proposed scheme has high detecting probabilities with lower computational complexity.

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