Multi-path Routing with End-to-end Statistical QoS Provisioning in Underlay Cognitive Radio Networks

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Abstract— **Since the radio access of secondary users is typically confined to ensure sufficient operation for primary users in underlay cognitive radio networks (CRNs), the inevitably induced latency and interference pose new challenges on existing routing schemes for Quality-of-Service (QoS) provisioning. Due to stringent accessing and interference constraints, secondary users appeal to exploit multi-path routing based on multi-hop relaying protocol to support QoS requirements. Via our model, we derive the end-to-end delay statistics including medium access and retransmission delay, where we successfully relate path diversity to end-to-end reliability and optimize the delivery delay by adjusting transmission power. We analyze the performance of the duplication-based and coding-aided multi-path routing schemes, where opportunistic transmission is employed to improve the delivery delay due to channel awareness, and encoding packets on multiple paths further achieves throughput efficiency. This paper firstly presents insights and performance analysis to facilitate multi-path routing with QoS provisioning in underlay CRN.**

I. INTRODUCTION

As a promising candidate of wireless networking paradigm, underlay cognitive radio networks (CRNs) [1], [2] emerge to enhance the spectrum utilization by realizing simultaneous transmissions of secondary users (SUs) with primary users (PUs) as long as sufficient operation of PUs is protected [3]. However, the induced interference and the confined radio access create challenges for the existing routing schemes to meet Quality-of-Service (QoS) requirements in such stringent environments. To facilitate end-to-end routing with QoS provisioning in underlay CRN, SUs appeal to exploit path diversity to achieve reliable multi-hop transmission [4], which is known as *multi-path routing*. Disjoint paths (i.e., the paths do not share any congestion points or bottlenecks) can be discovered via ad hoc on-demand multi-path distance vector routing protocol such as AMODV [5] based on the concept of link reversal extending from AODV. In tradition wireless ad hoc networks, erasure codes or forward error correction (FEC) are applied to provide low probability of packet loss by utilizing the path diversity (please refer to [6], [7] and the references therein for recent research). The source uses FEC to encode each packet into several fragments for multi-path transmission, and the reconstruction is possible if the destination receives sufficient fragments for decoding.

Distinct from traditional wireless ad hoc networks, the dynamic environment and confined radio access pose new threats on the QoS provisioning of underlay CRN. An SU suffers from severe interference from both PUs and other SUs, and the secondary transmissions must be confined to ensure sufficient operation of PUs, which thereby deteriorate the QoS provisioning. To overcome the technical difficulties, two multi-path routing schemes, *duplication-based multi-path routing* and *coding-aided multi-path routing*, are proposed based on multi-hop relaying protocol, where packets are decoded and stored in the buffer and then forwarded to the next hop when the SU is activated. Identical packets are forwarded on multiple paths for duplication-based multi-path routing, and encoding techniques are exploited for coding-aided multi-path routing to further achieve efficient multi-path transmission. We specifically exploit network coding [8] for coding-aided multi-path routing as a motivating example.

To investigate the performance of duplication-based and coding-aided multi-path routing schemes in underlay CRN, we focus on the analysis of the two most commonly adopted single-path routing schemes as the fundamental routing schemes, namely *path-predetermined routing* and *opportunistic routing*. For path-predetermined routing, packets are forwarded to the predetermined relay nodes based on the collected information at the source. A path is determined based on predicting available duration of secondary links according to the awareness of PUs in [9], and [10] chooses a path with the highest connectivity by evaluating the activity of PUs. On the other hand, packets are forwarded on the per-hop basis according to current channel status for opportunistic routing [11]. [12] proposes an opportunistic routing algorithm in CRN considering highly dynamic link availability features. However, there still lacks complete analytical model to characterize the end-to-end delay statistics in underlay CRN.

In this paper, we point out three main factors contributing to multi-hop routing latency: (i) hop number; (ii) retransmission; and (iii) medium access. Retransmission delay occurs due to outage events on secondary receiver (SR), medium access delay is affected by the medium access control protocol, and hop number further associates end-to-end delay with multi-hop transmission. Incorporating these factors, we first analyze the end-to-end delay statistics of single-path routing, and then we extend our model to duplication-based and coding-aided multi-path routing schemes. Moreover, we specifically implement network coding for coding-aided multi-path routing since the performance of network coding outweighs that of erasure codes in multi-hop wireless networks [13]. Our main contributions are that we provide a statistical end-to-end delay model for multi-path routing with QoS provisioning in underlay CRN by relating path diversity to end-to-end reliability, and we propose a novel mechanism to enhance the end-to-end throughput by encoding packets on multiple paths. To our best knowledge, this paper presents the first framework establishing analytical model of end-to-end delay statistics, providing useful insights and design guidelines for multi-path routing with QoS provisioning in underlay CRN.

II. SYSTEM MODEL

We consider the network model where SUs coexist with PUs, and each SU attempts to facilitate end-to-end communication while ensuring sufficient operation of PUs. To simplify the analysis, queueing delay is not considered here and perfect scheduling is assumed for multi-hop transmission. Based on previous effort [14], we leverage stochastic geometry to characterize the behavioral features of retransmission and medium access in underlay CRN. The spatial distributions of primary transmitters (PTs) and SUs are assumed to follow homogeneous Poisson point processes (PPPs) [15] with densities λ_{PT} and λ_{SU}, respectively. Each PT has transmission power P_{PT} and a dedicated primary receiver (PR) located at a fixed distance r_{PT} with an arbitrary direction. The spatial distribution of PRs also forms a PPP with the same density λ_{PT} correlated with that of PTs. Due to the stationary characteristics of PPP, the interference measured by the typical PR is representative of the interference seen by other PRs.

Since SUs transmit simultaneously with PUs in underlay CRN, it is an essential must for SUs to access the spectrum dynamically in order to meet the outage constraint of primary receiver sensitivity. To mitigate the interference to PRs, SUs adopt slotted ALOHA as the distributed spectrum access protocol for confined radio access. Each SU independently accesses the spectrum with probability $\widetilde{p}$ in each time slot, where $\widetilde{p}$ is the parameter of i.i.d. Bernoulli random variables, $B_i(\widetilde{p})$. The following lemma specifies the permissible active density of SUs adopting slotted ALOHA protocol.

Lemma 1: The permissible density of active SUs is $\widetilde{\lambda}_{SU} = \sigma P_{SU}^{-\delta}$*, and the active probability* $\widetilde{p} = \widetilde{\lambda}_{SU}/\lambda_{SU}$.

Proof: Let $\Phi_{PT} = \{X_i\}$ ($\Phi_{SU} = \{Y_i\}$) denote the locations of the PTs (SUs). The receiver sensitivity of a PR is maintained when only $\widetilde{p}$ portion of SUs are allowed to transmit simultaneously. This subset of SUs, denoted by $\Phi_{SU}(\widetilde{p}) = \{Y_i : B_i(\widetilde{p}) = 1\}$ with density $\widetilde{\lambda}_{SU} = \widetilde{p}\lambda_{SU}$, is obtained by independent thinning of Φ_{SU} with probability $\widetilde{p}$. We have the outage constraint on the typical PR as

$$\mathbb{P}\left(\frac{\mathcal{G}_{PT} P_{PT} r_{PT}^{-\alpha}}{N + I_{SU} + I_{PT}} \geq \eta_{PR}\right) = 1 - \epsilon_{PR}, \tag{1}$$

where ϵ_{PR} is the maximum outage probability imposed on PR, $\mathcal{G}_{PT}$ is the channel power gain of the desired link which is exponentially distributed with unit mean (i.e, slow flat Rayleigh fading channel), α is the path loss exponent, η_{PR} is the SINR threshold, N is the noise power level, $I_{SU} = \sum_{Y_i \in \Phi_{SU}(\widetilde{p})} \mathcal{G}_{Y_i} P_{SU} \|Y_i\|^{-\alpha}$ is the interference from SUs to the typical PR, $I_{PT} = \sum_{X_i \in \Phi_{PT}} \mathcal{G}_{X_i} P_{PT} \|X_i\|^{-\alpha}$ is the interference from other PTs to the typical PR, $\| \cdot \|$ denotes the distance to the typical PR, P_{SU} is the transmission power of SU and the channel power gain $\mathcal{G}_{X_i}$ and $\mathcal{G}_{Y_i}$ are also exponentially distributed with unit mean. From [14],

$$\begin{aligned} &\mathbb{P}\left[\mathcal{G}_{PT} \geq \frac{\eta_{PR}}{P_{PT} r_{PT}^{-\alpha}}(N + I_{SU} + I_{PT})\right] \\ &= \exp\left(-\frac{\eta_{PR}}{P_{PT} r_{PT}^{-\alpha}} N\right) \\ &\cdot \exp\left\{-\left(\widetilde{\lambda}_{SU}\left(\frac{P_{SU}}{P_{PT}}\right)^{\delta} + \lambda_{PT}\right) r_{PT}^{2} \eta_{PR}^{\delta} K_{\alpha}\right\}, \end{aligned} \tag{2}$$

and from (1) and (2), when $\frac{-\ln(1-\epsilon_{PR}) - \frac{\eta_{PR}}{P_{PT} r_{PT}^{-\alpha}} N}{r_{PT}^{2} \eta_{PR}^{\delta} K_{\alpha}} \geq \lambda_{PT}$,

$$\widetilde{\lambda}_{SU} = \left(\frac{-\ln(1-\epsilon_{PR}) - \frac{\eta_{PR}}{P_{PT} r_{PT}^{-\alpha}} N}{r_{PT}^{2} \eta_{PR}^{\delta} K_{\alpha}} - \lambda_{PT}\right)\left(\frac{P_{PT}}{P_{SU}}\right)^{\delta} \tag{3}$$

where $K_{\alpha} = \frac{2\pi^2}{\alpha \sin(2\pi/\alpha)}$, $\delta = 2/\alpha$ and $\widetilde{\lambda}_{SU} \triangleq \sigma P_{SU}^{-\delta}$. ■

A direct observation from *Lemma 1* is that the confined radio access of SUs in underlay CRN induces extra routing latency compared with traditional ad hoc networks. Nonetheless, based on the parametrization, we are able to derive the the end-to-end delay statistics of multi-path routing to support reliable end-to-end communications in underlay CRN. Throughout this paper, we choose the end-to-end delay as QoS measure, and a delivered packet is regarded as lost if the delivery delay exceeds some certain threshold. The statistical QoS provisioning is $P(\mathbf{T} \geq T_{th}) \leq \epsilon_T$, where $\mathbf{T}$ is the statistical end-to-end delay, T_{th} is the maximum tolerable delay and ϵ_T is the maximum outage probability of end-to-end delay. In addition, to measure the efficiency of multi-path routing, we define the end-to-end throughput as the number of different packets delivered per path for each end-to-end transmission. In other words, the end-to-end throughput is $C = \frac{N_P}{D}$, where N_P is the number of different packets and D is the number of end-to-end disjoint paths.

III. ANALYSIS OF SINGLE-PATH ROUTING

In this section, we derive the end-to-end delay statistics of path-predetermined and opportunistic routing schemes, serving as the foundation of multi-path routing. The main difference of the two routing schemes is that a packet is forwarded on a predefined route for the former one, whereas a packet is able to adjust route according to current channel status for the latter one. In addition, we formulate the end-to-end delay as an optimization problem, aiming to minimize the delivery delay by adjusting transmission power of SU.

As assumed in traditional wireless ad hoc networks [16], each SU is backlogged with its originated packets, and the relay nodes are separated by equal distance and placed on the line between source and destination to simplify the analysis and provide a lower bound on the delivery delay. Let L denote the distance between source and destination and H be the hop number. The relay nodes are separated by the same distance $r_{SU} = L/H$, and thus all the links of the multi-hop path have the same reliability. Since the hop number of an end-to-end path can be acquired by adopting route discovery mechanism, the end-to-end delay on a single path with H hops is interpreted as the time required to attain H successes with probability of successful q for each trial, which is derived in the subsequent paragraph.

Lemma 2: The end-to-end delay of single-path routing, $\mathbf{T}_S$*, is a negative binomial random variable* $\mathrm{NB}(H, q)$ *with* H *successful transmissions and probability of success* q*.*

Proof: Adopting slotted ALOHA protocol, the number of trials (time slots) needed for an SU to transmit a packet successfully to the next hop is a geometric random variable with probability q. The end-to-end delay $\mathbf{T}_S$ is hence the sum of H i.i.d geometric random variables, i.e., a negative binomial random variable $\mathrm{NB}(H, q)$. ■

Moreover, when adopting slotted ALOHA protocol, the average buffer occupancy of a delivered packet is the time for an SU to successfully forward the packet to the next SU.

Corollary 1: For a delivered packet, the average buffer occupancy of a relaying SU is $\frac{1}{q}$ *time slots.*

Incorporating medium access and retransmission delay, we obtain the parameter $q = \widetilde{p} \cdot p^s$, where $\widetilde{p}$ is the active probability of an SU derived in *Lemma 1* and p^s is the probability of successful reception at an SR. In the sequel we explicitly derive p^s for both path-predetermined and opportunistic routing schemes. For path-predetermined routing, we have

$$\begin{aligned} p^s_{pre} &= \mathbb{P}\left[\mathcal{G}_{SU} \geq \frac{\eta_{SU}}{P_{SU} r_{SU}^{-\alpha}}(N + I_{PT} + I_{SU})\right] \\ &= \exp\left(-\frac{\eta_{SU} N}{P_{SU}} r_{SU}^{\alpha}\right) \\ &\quad \cdot \ \exp\left\{-\left(\lambda_{PT}\left(\frac{P_{PT}}{P_{SU}}\right)^{\delta} + \widetilde{\lambda}_{SU}\right) r_{SU}^2 \eta_{SU}^{\delta} K_{\alpha}\right\} \end{aligned} \tag{4}$$

where $\mathcal{G}_{SU}$ denotes the channel power gain of the desired link which is exponentially distributed with unit mean, r_{SU} is the transmission distance of SU and η_{SU} is the SINR threshold. Substituting $r_{SU} = L/H$ and $\widetilde{\lambda}_{SU} = \sigma P_{SU}^{-\delta}$, we obtain

$$\begin{aligned} p^s_{pre} &= \exp\left(-\frac{\eta_{SU} N}{P_{SU}}\left(\frac{L}{H}\right)^{\alpha}\right) \\ &\quad \cdot \ \exp\left\{-\left(\lambda_{PT}\left(\frac{P_{PT}}{P_{SU}}\right)^{\delta} + \sigma P_{SU}^{-\delta}\right)\left(\frac{L}{H}\right)^2 \eta_{SU}^{\delta} K_{\alpha}\right\} \\ &= \exp\left\{-k_1 P_{SU}^{-1} H^{-\alpha} - (k_2 + k_3) P_{SU}^{-\delta} H^{-2}\right\}, \end{aligned} \tag{5}$$

where $k_1 = \eta_{SU} N L^{\alpha}$, $k_2 = \lambda_{PT} P_{PT}^{\delta} L^2 \eta_{SU}^{\delta} K_{\alpha}$, and $k_3 = \sigma L^2 \eta_{SU}^{\delta} K_{\alpha}$.

On the other hand, the channel awareness of opportunistic routing enables an SU transceiver pair active if its current channel gain is above some threshold η_{OR} so that $\widetilde{p}$ fraction of SU transceiver pairs with the highest channel gains are activated. Here we assume that opportunistic routing always selects the concurrent route with highest channel gain, and the relay hop is of equidistant, which is consistent with [17].

Lemma 3: The channel power gain of opportunistic routing, $\mathcal{G}_{OR}$*, is an exponential random variable with unit mean after its distribution function is shifted by* $\ln\frac{1}{\widetilde{p}}$*.*

Proof: Considering channel status, an SU is activated only when its channel power gain is larger than some threshold $\eta_{OR} \geq 0$. Since the outage constraint of primary receiver sensitivity in *Lemma 1* must be satisfied, we have $\mathbb{P}(\mathcal{G}_{SU} \geq \eta_{OR}) = \widetilde{p}$ and thus obtain $\eta_{OR} = -\ln\widetilde{p}$. The complementary cumulative distribution function (CCDF) of $\mathcal{G}_{OR}$ is

$$\begin{aligned} \bar{F}_{\mathcal{G}_{OR}}(x) &= \mathbb{P}(\mathcal{G}_{OR} \geq x) = \mathbb{P}(\mathcal{G}_{SU} \geq x \mid \mathcal{G}_{SU} \geq \eta_{OR}) \\ &= e^{-(x - \eta_{OR})}, \ x \geq \eta_{OR}, \end{aligned} \tag{6}$$

which is exactly the CCDF of an exponential random variable with unit mean after it is shifted by $\ln\frac{1}{\widetilde{p}}$. ■

In the interference-limited regime (neglect the background noise N), by generalizing the results of [18] to heterogeneous case, we have

$$\begin{aligned} p^s_{OR} &= \mathbb{P}\left[\mathcal{G}_{OR} \geq \frac{\eta_{SU}}{P_{SU} r_{SU}^{-\alpha}}(N + I_{PT} + I_{SU})\right] \\ &\overset{N=0}{\approx} \exp\left\{-\left(\lambda_{PT}\left(\frac{P_{PT}}{P_{SU}}\right)^{\delta} + \widetilde{\lambda}_{SU}\right)\pi r_{SU}^2 \eta_{SU}^{\delta}\right. \\ &\quad \left. \cdot \ \mathbb{E}[\mathcal{G}^{\delta}]\mathbb{E}[\mathcal{G}_{OR}{}^{-\delta}]\right\} \\ &= \exp\left\{-\left(\lambda_{PT} P_{PT}^{\delta} + \sigma\right) P_{SU}^{-\delta} \pi \left(\frac{L}{H}\right)^2 \eta_{SU}^{\delta}\right. \\ &\quad \left. \cdot \ \Gamma(1 + \delta, 0)\Gamma(1 - \delta, \eta_{OR})\right\}, \end{aligned} \tag{7}$$

where $\Gamma(a, b) = \int_b^{\infty} x^{b-1} e^{-x} dx$ is the upper incomplete gamma function, and $\mathcal{G}$ is the channel power gain of interfering links which is exponentially distributed with unit mean.

Moreover, by incorporating the parameters derived in *Lemma 1* and *Lemma 3* into *Lemma 2*, the mean end-to-end delay of path-predetermined and opportunistic routing are shown in the following corollary due to the fact that $\mathbf{T}_S$ is a negative binomial random variable.

Corollary 2: $\mathbb{E}[\mathbf{T}_{pre}] = \frac{H}{\widetilde{p} p^s_{pre}}$ *and* $\mathbb{E}[\mathbf{T}_{OR}] = \frac{H}{\widetilde{p} p^s_{OR}}$.

The mean end-to-end delay of opportunistic routing is smaller than that of path-predefined routing since $p^s_{OR} > p^s_{pre}$. From *Corollary 2*, the mean end-to-end delay of single-path routing, $\mathbb{E}[\mathbf{T}_s]$, can be formulated as a delay-minimizing problem with respect to SU transmission power P_{SU}. For path-predetermined routing, the optimization problem is

$$\begin{aligned} &\text{Minimize } H k_4 P_{SU}^{\delta} \exp\left\{k_1 P_{SU}^{-1} H^{-\alpha} + (k_2 + k_3) P_{SU}^{-\delta} H^{-2}\right\} \\ &\text{Subject to } H \geq 1, \ P_{SU} \geq 0, \end{aligned} \tag{8}$$

where $k_4 = \lambda_{SU} \sigma^{-1}$. Since H can be obtained by route discovery and $\mathbb{E}[\mathbf{T}_{pre}]$ is a convex function of P_{SU} when H is known, the optimal transmission power is obtained by differentiating (8) with respect to P_{SU} and setting the result equal zero. Similarly, we can solve the optimal SU transmission power of opportunistic routing to minimize the mean end-to-end delay.

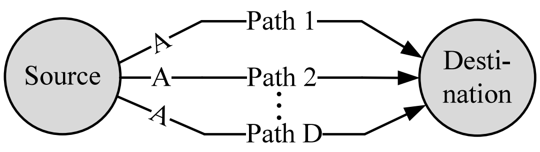

Fig. 1. Illustration of Duplication-based multi-path routing. Packet A is delivered on D disjoint multi-hop paths. The end-to-end throughput is apparently $C_{dup} = \frac{1}{D}$.

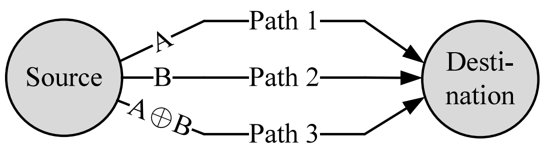

Fig. 2. Illustration of coding-aided multi-path routing via network coding for $D = 3$. Delivering the encoded packet $A \oplus B$ provides fractional path diversity for both packets A and B. The end-to-end throughput is $C_{cod} = \frac{2}{3}$.

IV. DUPLICATION-BASED MULTI-PATH ROUTING

In this section, we extend the end-to-end delay statistics of single-path routing schemes to duplication-based multi-path routing, where identical data are delivered through multiple disjoint paths to improve the delivery delay as illustrated in Fig. 1. We assume there are D disjoint paths of distance L between source and destination, and the transmission power of SU on each path is the solution of the delay-minimizing problem (8).

Proposition 1: The end-to-end delay of duplication-based multi-path routing, $\mathbf{T}_M$, *is a random variable with CDF* $\bar{F}_{\mathbf{T}_M}(t) = \prod_{i=1}^{D} \left[1 - \sum_{j=H_i}^{t} \binom{j-1}{H_i-1}(1-q)^{j-H_i} q^{H_i}\right]$, *where t is an integer in the unit of time slot and* $t \ge \min\{H_i\}$.

Proof: Denoting $\mathbf{Z}_i$ the end-to-end delay and H_i the hop number of ith path, from *Lemma 2* we have $\mathbf{Z}_i \sim \mathrm{NB}(H_i, q)$. The delivery delay utilizing D disjoint paths is thus $\mathbf{T}_M = \min\{\mathbf{Z}_1,\ \mathbf{Z}_2, \dots,\ \mathbf{Z}_D\}$. Knowing $\mathbf{Z}_i \ge H_i$, we have

$$F_{\mathbf{Z}_i}(z_i) = \mathbb{P}(\mathbf{Z}_i \le z_i) = \sum_{j=H_i}^{z_i} \binom{j-1}{H_i-1}(1-q)^{j-H_i} q^{H_i} \tag{9}$$

By (9), the CCDF of $\mathbf{T}_M$ is

$$\begin{aligned} \bar{F}_{\mathbf{T}_M}(t) &= \mathbb{P}(\mathbf{T}_M > t) = \mathbb{P}(\min\{\mathbf{Z}_1,\ \mathbf{Z}_2, \dots,\ \mathbf{Z}_D\} > t) \\ &= \mathbb{P}(\mathbf{Z}_1 > t, \mathbf{Z}_2 > t, \cdots, \mathbf{Z}_D > t) = \prod_{i=1}^{D} \mathbb{P}(\mathbf{Z}_i > t) \\ &= \prod_{i=1}^{D} \left[1 - \sum_{j=H_i}^{t} \binom{j-1}{H_i-1}(1-q)^{j-H_i} q^{H_i}\right] \end{aligned} \tag{10}$$

for $t \ge \min\{H_i\}$. And the mean end-to-end delay is

$$\begin{aligned} \mathbb{E}[\mathbf{T}_M] &= \sum_{t=\min\{H_i\}}^{\infty} \mathbb{P}(\mathbf{T}_M > t) \\ &= \sum_{t=\min\{H_i\}}^{\infty} \prod_{i=1}^{D} \left[1 - \sum_{j=H_i}^{t} \binom{j-1}{H_i-1}(1-q)^{j-H_i} q^{H_i}\right] \end{aligned} \tag{11}$$

Note that $\mathbf{T}_S$ is a degenerate case of $\mathbf{T}_M$ when $D = 1$. ■

Proposition 1 is a generic model including paths with distinct hop numbers and different routing schemes. The mean end-to-end delay of duplication-based multi-path routing via path-predetermined routing (opportunistic routing) can be obtained by setting $q = \widetilde{pp}^{s}_{pre}$ ($q = \widetilde{pp}^{s}_{OR}$). More importantly, with the aid of our model, we are able to calculate the path diversity required for QoS provisioning since the end-to-end delay statistics of multi-path routing are available, which offers new avenues to routing designs in underlay CRN.

V. CODING-AIDED MULTI-PATH ROUTING

To fully exploit the multiple end-to-end paths, we implement network coding techniques [8] at the source to enhance the end-to-end throughput while guaranteeing the QoS provisioning of each original packet. Instead of delivering duplicated data, the source encodes the contents of the packets from its buffer by the field operation $\oplus$ and deliver the encoded packets on disjoint paths to achieve efficient transmission as shown in Fig. 2. The end-to-end delay of an original packet is determined by the criterion that the destination has to receive sufficient data for successful decoding.

Proposition 2: The end-to-end delay of an original packet is the time the receiver received sufficient encoded packets on D disjoint paths to successfully decode the packet.

Intuitively, the end-to-end throughput can be further enhanced via network coding since different packets are able to fractionally utilize the same path by delivering the encoded packet, provided that the QoS provisioning of each original packet is still satisfied. In other words, network coding is leveraged to tune the QoS provisioning of the delivered packets supported by multi-path routing. Take Fig. 2 as an example, suppose that single-path routing fails to support the QoS provisioning of packets A and B while using two paths may overprovide the QoS provisioning. Instead of forwarding packet A and packet B separately via duplication-based multi-path routing (i.e., the end-to-end throughput is $C_{dup} = \frac{1}{2}$), it is possible to encode the two packets on path 3 to enhance the end-to-end throughput if the QoS provisioning is still sufficient, and the end-to-end throughput becomes $C_{cod} = \frac{2}{3}$. Consequently, packets A and B possess fractional path diversity on path 3 since the two packets are delivered concurrently.

To demonstrate our idea, we explicitly derive the end-to-end delay statistics of packets A and B in Fig. 2. The delivery delay of packet A is the time required for the destination to receive the data from either path 1 or both path 2 and path 3. The end-to-end delay of packet A, $\mathbf{T}_A$, is

$$\mathbf{T}_A = \min\{\mathbf{T}_1, \max\{\mathbf{T}_2, \mathbf{T}_3\}\}. \tag{12}$$

From *Lemma 2*, we have

$$\begin{aligned} &\mathbb{P}\left(\max\{\mathbf{T}_2, \mathbf{T}_3\} \le t\right) \\ &= \mathbb{P}\left(\{\mathbf{T}_2 \le t\} \cap \{\mathbf{T}_3 \le t\}\right) \\ &= \mathbb{P}\left(\mathbf{T}_2 \le t\right) \mathbb{P}\left(\mathbf{T}_3 \le t\right) \\ &= \prod_{i=2}^{3} \left(\sum_{j=H_i}^{t} \binom{j-1}{H_i-1}(1-q)^{j-H_i} q^{H_i}\right). \end{aligned} \tag{13}$$

And we obtain the CCDF of $\mathbf{T}_A$ as

$$\begin{aligned}\bar{F}_{\mathbf{T}_A}(t) &= \mathbb{P}\left(\mathbf{T}_A > t\right) = \mathbb{P}\left(\min\left\{\mathbf{T}_1, \max\{\mathbf{T}_2, \mathbf{T}_3\}\right\} > t\right)\\ &= \mathbb{P}\left(\{\mathbf{T}_1 > t\} \cap \{\max\{\mathbf{T}_2, \mathbf{T}_3\} > t\}\right)\\ &= \mathbb{P}\left(\mathbf{T}_1 > t\right)\mathbb{P}\left(\max\{\mathbf{T}_2, \mathbf{T}_3\} > t\right)\\ &= \left(1 - \sum_{j=H_1}^{t} \binom{j-1}{H_1-1}(1-q)^{j-H_1} q^{H_1}\right)\\ &\cdot \left[1 - \prod_{i=2}^{3}\left(\sum_{j=H_i}^{t}\binom{j-1}{H_i-1}(1-q)^{j-H_i} q^{H_i}\right)\right] \quad (14)\end{aligned}$$

Similarly, we also obtain the CCDF of $\mathbf{T}_B$ as

$$\begin{aligned}\bar{F}_{\mathbf{T}_B}(t) &= \mathbb{P}\left(\mathbf{T}_B > t\right) = \mathbb{P}\left(\min\left\{\mathbf{T}_2, \max\{\mathbf{T}_1, \mathbf{T}_3\}\right\} > t\right)\\ &= \left(1 - \sum_{j=H_2}^{t} \binom{j-1}{H_2-1}(1-q)^{j-H_2} q^{H_2}\right)\\ &\cdot \left[1 - \prod_{i=1, i\neq 2}^{3}\left(\sum_{j=H_i}^{t}\binom{j-1}{H_i-1}(1-q)^{j-H_i} q^{H_i}\right)\right] \quad (15)\end{aligned}$$

Based on the end-to-end delay statistics, we are able to build up a lookup table of end-to-end delay to check whether the QoS provisioning of each original packet is satisfied via coding-aided multi-path routing. Note that if packet A is transmitted on path 3 instead of the encoded packet $A \oplus B$, the end-to-end throughput is still $C_{cod} = \frac{2}{3}$, but the corresponding end-to-end delay of packet B reduces to $\mathbf{T}_B = \mathbf{T}_2$, which deteriorates the QoS provisioning of packet B. Moreover, the feature of fractional path diversity is quite straightforward that

$$\mathbb{P}(\mathbf{T}''_A > t) \leq \mathbb{P}(\mathbf{T}_A > t) \leq \mathbb{P}(\mathbf{T}'_A > t), \quad (16)$$

where $\mathbf{T}''_A$ ($\mathbf{T}'_A$) is the end-to-end delay of the duplication-based multi-path routing when $D = 2$ ($D = 1$). Suppose that $\mathbf{T}_A$ satisfies the QoS provisioning, we are able to deliver packet B with packet A via network coding since packet A requires only fractional path diversity gain from path 3 (i.e. the overall path diversity of packet A is between one and two), and the QoS provisioning of packet B benefits from the remaining fractional path diversity from path 3. Coding-aided multi-path routing therefore provides novel mechanisms for the tradeoffs between QoS provisioning and end-to-end throughput by utilizing fractional path diversity.

VI. PERFORMANCE EVALUATION

Following [14], the system parameters are set to be $N = 10^{-9}$mW, $\alpha = 4$, $\eta_{SU} = 3$, $\epsilon_{SU} = 0.1$, $\lambda_{PT} = 10^{-5}$PUs/m^2, $P_{PT} = 0.3$mW, $r_{PT} = 15$m, $\eta_{PR} = 3$, $\epsilon_{PR} = 0.05$ and $L = 200$m on a 1000×1000m^2 square field. The slot time is 1ms and the hop number is assumed to be equal for all disjoint paths, i.e., $H_i = H$. Moreover, the mean end-to-end delay of all routing schemes are minimized by adopting the optimal transmission power obtained in (8).

We compare the performance of multi-hop transmission via single-path and multi-path routing schemes in Fig. 3. The mean end-to-end delay of all routing schemes are mitigated via multi-hop transmission, but it also possesses marginal gain when the hop number is large. Opportunistic routing (abbreviated as OR) greatly mitigates the delivery delay by reducing the possible retransmissions due to channel awareness compared with path-predetermined routing (abbreviated as PRE). More interestingly, OR outperforms duplication-based multi-path path-predetermined routing (abbreviated as DM-PRE) for $D = 3$ in multi-hop transmission, suggesting that DM-PRE takes advantage of path diversity for direct (one-hop) transmission, whereas OR advances in multi-hop transmission since the packet is forwarded to the next hop with higher channel power gain. Furthermore, duplication-based multi-path opportunistic routing (abbreviated as DM-OR) further improves the end-to-end delay since it benefits from both path diversity and channel awareness.

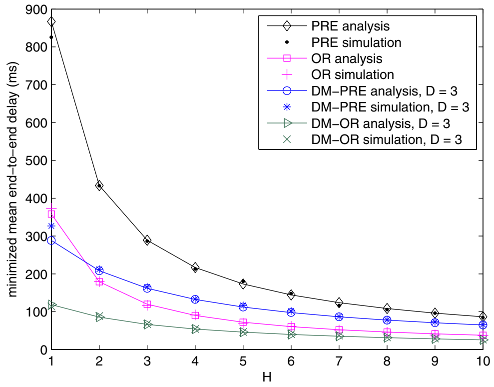

Fig. 3. Minimized mean end-to-end delay with respect to hop number for $\lambda_{SU} = 5 \cdot 10^{-4}$ SUs/m^2.

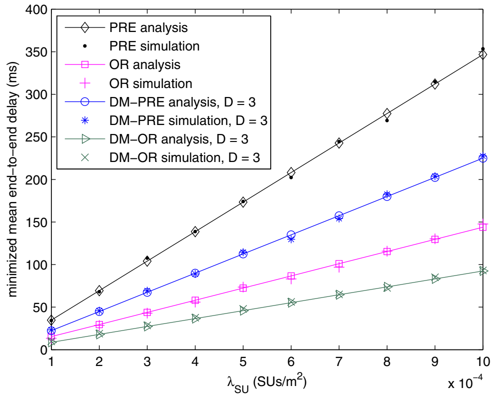

Fig. 4. Minimized mean end-to-end delay with respect to SU density for $H = 5$.

In addition, the end-to-end delay exhibits linear scalability with respect to SU density as shown in Fig. 4. The results are quite reasonable due to the fact that the radio access of SUs are confined to ensure sufficient operation of PUs as discussed in Section II, and the end-to-end delay increases with the decrease of active probability. Similar results are found for the average buffer occupancy of a relaying SU

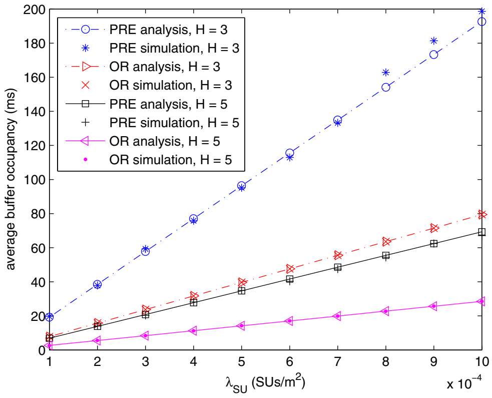

Fig. 5. Average buffer occupancy of a relaying SU with respect to SU density.

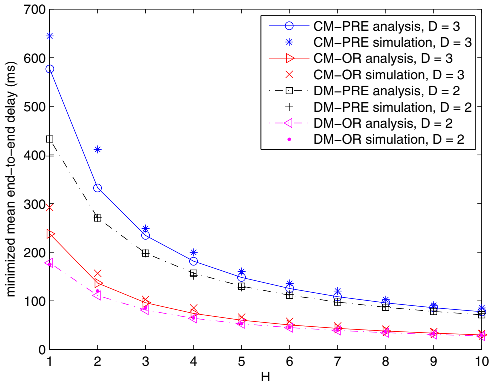

Fig. 6. Minimized mean end-to-end delay of packet A in Fig. 2 with respect to hop number for $\lambda_{SU} = 5 \cdot 10^{-4}$ SUs/m^2. Coding-aided multi-path path-predetermined (opportunistic) routing via networking is abbreviated as CM-PRE (CM-OR).

as shown in Fig. 5. Increasing hop number reduces average buffer occupancy due to enhancement of link reliability. OR shortens the buffer occupancy since higher channel power gain reduces the possible retransmissions, and the average buffer occupancy also exhibits linear scalability with respect to SU density due to growing population. Finally, the performance of implementing network coding for coding-aided multi-path routing is shown in Fig. 6. By utilizing multi-hop transmission, the end-to-end delay of packet A (as well as B) is comparable with that of the duplication-based multi-path routing schemes of $D = 2$ when the hop number is large. These results therefore offer novel avenues to QoS provisioning and end-to-end throughput enhancement in underlay CRN.

VII. CONCLUSION

Summarizing this paper, we firstly provide a mathematical tool relating path diversity to end-to-end reliability for QoS provisioning in underlay CRN. Considering the impacts of interference and dynamic spectrum access, we derive the end-to-end delay statistics including medium access and retransmission delay and solve the optimal transmission power to minimize the delivery delay. The end-to-end delay and average buffer occupancy benefit from channel awareness as well as path diversity by opportunistic transmission and forwarding duplicated packets. Moreover, coding-aided approach provides fractional path diversity by delivering encoded packets on multiple paths, and the results show that implementing network coding further enhances the end-to-end throughput without deteriorating the QoS provisioning of each original packet via multi-hop transmission. This paper therefore provides significant insights and performance evaluation to facilitate multi-path routing with QoS provisioning in underlay CRN.

VIII. ACKNOWLEDGEMENT

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