Proportionally Fair Rate Allocation in Regular Wireless Sensor Networks

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***Abstract*—In this paper, we consider the problem of fair rate allocation that maximizes the network throughput in regular topologies of Wireless Sensor Networks (WSNs). In order to monitor the entire coverage area of the WSN while maintaining acceptable network throughput, we need to find the optimal rate allocation for the individual/competing end-to-end sessions that maximizes the total proportionally fair throughput of the network. We provide closed form expressions for the optimal end-to-end session rates for square, triangular and hexagonal topologies as well as the bounds for the link layer transmission probabilities. We study the aforementioned problem for regular WSNs with a slotted Aloha MAC layer, which provides us with a lower bound for more realistic MAC protocols. Real world experiments using Telosb nodes validate our theories and results. Simulations carried out in Qualnet verify our comparisons of the different regular topologies as the size of network grows.**

I. Introduction

The node deployment problem in Wireless Sensor Networks (WSNs) deals with strategies for the placement of individual sensor nodes (SNs), such that the resulting network satisfies specified constraints. A popular method of deploying WSNs is to scatter the sensor nodes randomly across the area to be monitored. However, this kind of random topology is not necessarily effective or efficient, as there could not only be areas that are not covered by any SN, but also several redundant nodes that do not contribute towards enhancing the network performance [1]. The idea of sleep scheduling is applied to WSNs to make better use of extra SNs. An alternate approach to handling redundancy, while also reducing the cost of node deployment, is to deploy the sensor nodes in a regular topology.

Regular node deployment strategies have been shown to minimize the number of nodes required to completely cover a given area in two dimensional space, while maintaining connectivity across the network [2]. Another benefit of a regular network is its simplicity. Ease of deployment and maintenance is an attractive feature for commercial applications, involving WSNs in physically accessible areas, that aim to optimize the performance of the network. Many works related to regular topologies exist and most of them deal with optimizing the energy efficiency, coverage, connectivity, throughput, or a combination of them [3] [4].

Given the emphasis on eliminating redundancy, it is imperative that each SN in a regular WSN be able to adequately send its monitored data to the Base Station (BS). This calls for a rate control scheme that is fair to all the nodes (sessions originating in them) in the network. Absolute fairness among competing sessions, however, may cause the total throughput of the network to significantly decrease, rendering the performance of the network unacceptably poor [5]. Hence, we study the allocation of session rates to maximize the throughput of the WSN, while maintaining proportional fairness in the network [6].

In this work, we consider the problem of end-to-end proportionally fair rate allocation in random access WSNs, deployed in the three most common regular lattice topologies namely, the square, hexagonal, and triangular. We adopt a cross layer solution approach where the session rates at the transport layer and the MAC layer link capacities are inter-dependent, based on the non-convex formulation presented in [6]. Our main contributions in this paper are as follows:

- For each of the three regular topologies, we derive optimal session rates and optimal MAC transmission probabilities that maximizes the total proportionally fair throughput.
- Closed form expressions for the total throughput of each regular WSN, in the context of proportional fairness, are presented in Table I. Based on the expressions, we are able to conclude that as the network size grows from small, to medium, to large, the topology with the best proportionally fair throughput changes from triangular, to square, to hexagonal.
- Experiments conducted using TELOSB motes support our theoretical findings, as well as highlight some important challenges faced while applying theoretical results to the real world wireless medium.
- We perform extensive simulations to verify our results for larger WSNs.

This paper is organized as follows. Section II describes other works related to regular topologies and proportional fairness. In Section III, we describe our system model and formulate the rate allocation problem. In Section IV, we provide our solution approach, that demonstrates certain key characteristics of proportionally fair session rate control. This is augmented by a numerical analysis of our solution which reveals the behaviour of different regular topologies at different network

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sizes. Section V explains the set up and outcome of experiments carried out on TELOSB motes. In Section VI, we present the results of our simulations conducted in Qualnet. Section VII concludes the paper and discusses directions of future work.

II. Related Work

In this section, we describe the existing works that deal with regular topologies of WSNs and also the fair rate allocation problem. Y.Wang et al. [2] analyse the different regular WSN deployments (square, triangular and hexagonal lattices) in terms of the minimum number of sensors required to achieve full coverage and connectivity. Another work related to the maximization of network lifetime of grid-based WSNs is given in [7]. The proportionally fair rate allocation problem for general wireless networks is the focus of Wang and Kar's work in [6]. They provide a distributed polynomial time solution for the proportionally fair rate allocation problem based on cross-layer optimizations, by introducing a "co-operation" between the link layer transmission probabilities and transport layer session rates. They evaluate the performance of their algorithms on small scale wireless networks. We aim to exploit the characteristics of regular WSNs, in order to compute closed form expressions for the maximum proportionally fair network throughput, as well as for the end-to-end session rates and transmission probabilities to achieve it. We hope to use this to determine the performance of the different regular topologies at different network sizes. In [8], Liu et.al compare the per-node throughput (or capacity) of individual nodes, and end-to-end throughput(defined as the minimum per-node throughput of all nodes considered) of regular WSNs using a slotted Aloha MAC. They do not, however, consider competing end-to-end sessions or proportional fairness in their analysis.

III. Problem Formulation

We consider the end-to-end proportionally fair rate allocation problem in a wireless sensor network, where wireless sensor nodes are placed in a regular topology with a base station (BS) at the center of the network.

We model the wireless network as an undirected graph $G = (V, E)$. V represents the set of nodes, and E is the set of undirected edges connecting the nodes of G. Let $n = |V \backslash \{BS\}|$, represent the total number of nodes in the network, excluding the BS. Directed link $(i, j) \in E$ represents an active link from node i to j, and L is the set of all active paths in the network. We specify, for each node i, its neighbour as N^i. The regular topology means that every node i has the same number of neighbouring nodes $|N^i|$. Additionally, sets O^i and I^i are the outgoing and incoming neighbours of node i, defined on the basis of its outgoing and incoming active links. The size of a regular topology, that is the maximum hop distance from BS to the nodes on the boundary, scales incrementally from 1 to K. Table I gives the total number of nodes excluding the BS in the three regular topologies of size K. Every node continuously monitors the environment, and periodically forwards all its collected data to the BS. There is no data aggregation and the BS is just a sink node. The transport layer sessions are represented by the set S. Also, $S(i, j)$ represents the set of sessions on link (i, j) that belongs to S and $|S(i, j)|$ represents the size of that set. We assume a routing scheme that evenly balances the load experienced by each 1-hop neighbour of the BS, which means that the number of sessions that belongs to links (v, BS) and (w, BS) are equal, where $v, w \in N^{BS}$ and $v \neq w$. Load balancing is typically true of any WSN, since maximizing the lifetime is their common goal. The MAC layer uses a slotted aloha scheme, which would serve as a useful lower bound for more realistic MAC protocols [3]. The MAC layer model from [6] is used in our analysis. We present that model here for completeness. Each node has a probability P_i of transmitting in a slot, and $p_{i,j}$ is the access probability to transmit to a particular outgoing link (i, j). Then, we can define the capacity (rate of any link) as,

$$x_{i,j} = c_{i,j} = p_{i,j}\left(1 - P_j\right) \prod_{k \in N^j/\{i\}} 1 - P_k$$

A. Problem Statement

For each regular topology, we determine both the optimal session rate y_s^*, for each session $s \in S$, and the optimal transmission probabilities $p_{i,j}^*$, that maximizes the total proportionally fair session rate, through cross layer rate control. The proportionally fair session rate is represented by the logarithmic utility function, and the total proportionally fair session rate is defined by their summation. The original non-convex formulation of this problem for any network topology is stated by Wang and Kar [6]. Next, we also derive closed form expressions which reveal the proportionally fair rate allocation characteristics of each regular topology at different scales.

Finally, we support the following statement by comparing the three equations for each regular topology. *The optimal proportionally fair rates can be achieved at small, medium, and large scale with triangular, square, and hexagonal topology, respectively*.

IV. Proportionally Fair Allocation for Regular Topology

We first determine y_s^* and $p_{i,j}^*$ for the square topology. The same procedures can be easily applied to the other regular topologies, since y_s^* depends only on the $p_{i,BS}^*$, $i \in N^{BS}$, which is proved in Lemmas 4.1 and 4.2.

Since there is no benefit to not fully utilizing the capacity of the links, (i, BS), $\forall i \in N^{BS}$, under the load balancing routing scheme, it is reasonable to believe that $p_{i,BS} = q$, $\forall i \in N^{BS}$, where $0 < q < 1$. In fact, $p_{i,BS}^* = q^* = 0.25$. This can be easily determined by solving for maximum q by taking the derivative of $c_{i,BS} = q(1-q)^3$. Since the BS is just a sink node, $P_{BS} = 0$. We denote this simple fact as Principle 1.

Highest data traffic load is experienced by N^{BS} as compared to any other node in the network. Therefore, intuitively, it is fair to allocate lower session rates to the 1-hop nodes, than

Topologies	n	$y^*_s, \forall s \in S$	$\sum_{\forall s \in S} \log y^*_s$
Hexagonal	$\frac{3}{2}\left(K+K^2\right)$	$2\frac{\frac{1}{3}\left(\frac{2}{3}\right)^2}{K+K^2}$	$\frac{3}{2}\left(K+K^2\right)\log\left(2\frac{\frac{1}{3}\left(\frac{2}{3}\right)^2}{K+K^2}\right)$
Square	$2\left(K+K^2\right)$	$2\frac{\frac{1}{4}\left(\frac{3}{4}\right)^3}{K+K^2}$	$2\left(K+K^2\right)\log\left(2\frac{\frac{1}{4}\left(\frac{3}{4}\right)^3}{K+K^2}\right)$
Triangular	$3\left(K+K^2\right)$	$2\frac{\frac{1}{6}\left(\frac{5}{6}\right)^5}{K+K^2}$	$3\left(K+K^2\right)\log\left(2\frac{\frac{1}{6}\left(\frac{5}{6}\right)^5}{K+K^2}\right)$

TABLE I
PROPORTIONALLY FAIR SESSION RATES AND OPTIMAL CHANNEL ACCESS PROBABILITIES FOR DIFFERENT REGULAR TOPOLOGIES AT DIFFERENT SCALES, K.

those allocated to nodes farther away from the BS. However, surprisingly, this is not true in case of proportionally fair rate allocation in regular topologies. We prove this fact next.

Lemma 4.1: To maximize the total proportional fairness of the network, the individual session rates y_s, $s \in S$, should all be equal.

Proof: Every session, regardless of its rate y_s, $s \in S$, must be forwarded to the BS to ensure that the WSN is not disconnected. Also, every session experiences the same amount of interference at the BS. Suppose, we index each session as $s = \{1, 2, \ldots, n\}$. Assume that the links to the nodes that are 1 hop away from the BS, (i, BS), $\forall i \in N^{BS}$, are the bottleneck for all the sessions in S. Each session $s \in S(i, BS)$, $\forall i \in N^{BS}$, would maximize its rate y_s until the maximum capacity $c^*_{i,BS}$, set by Principle 1, is reached. Therefore, the summation of the optimal session rates y^*_s is bounded, and so there exists a unique constant y such that,

$$\sum_{i=1}^{n} (y^*_i) = ny.$$

We prove by contradiction that the maximum value of the total logarithmic session rate, when each session rate is distinct, is not greater than when all session rates are equal. Suppose,

$$k \log (y) \leq \sum_{i=1}^{k} \log (y_i). \tag{1}$$

Then, the following must also be true.

$$n \log \left(\frac{1}{n} \sum_{i=1}^{n} (y^*_i) \right) \leq \sum_{i=1}^{n} \log (y^*_i)$$

But, based on the Arithmetic-Geometric mean inequality,

$$\left(\frac{1}{n} \sum_{i=1}^{n} (y^*_i) \right)^n > \prod_{i=1}^{n} y^*_i,$$

which contradicts Eqn. 1. ■

Our next Lemma proves that (i, BS), $\forall i \in N^{BS}$, indeed serve as the bottleneck for the entire network, in case of the square topology, and this only weakly depends on P_j, $j \in V \backslash \{N^{BS}, BS\}$.

Lemma 4.2: The optimal allocation of rates to the individual sessions, $s \in S$, is constrained by the capacity of the 1-hop links, $c_{i,BS}$, $\forall i \in N^{BS}$, which serve as the bottleneck for the entire network. This fact is only weakly dependent on P_k, where $2 \leq k \leq K$.

Proof: First, we define the Link Utilization Factor of a session, $s \in S(i,j)$, on link (i,j), as $U_{i,j}(s)\Big|_{s \in S(i,j)} = \frac{c_{i,j}}{|S(i,j)|}$. This represents the bandwidth allocated to each session on the link (i,j). Due to the symmetry of the regular topologies and load balanced routing scheme, each link that is k hops away from the BS, supports approximately an equal number of sessions. Therefore, the link access probability, P_k, of a node at k hops away from the BS, should be equal for all nodes at a distance of k-hops (a similar argument was used in Principle 1). In the square topology, the number of sessions on link $(k-hop, (k-1)-hop)$ is $\frac{(K+K^2)-k(k-1)}{2k}$. For the sake of convenience, we will drop the word '*hop*' and just refer to the link that is k hops from the BS as $(k, k-1)$. Then,

$$U^*_{1,BS}(s)\Big|_{s \in S(1,BS)} = \frac{2c^*_{1,BS}}{K+K^2} \tag{2}$$

$$U_{k,(k-1)}(s)\Big|_{s \in S(k,(k-1))} = \frac{2kc_{k,(k-1)}}{K+K^2-k(k-1)}, \tag{3}$$

where $c^*_{1,BS} = q^*(1-q^*)^3$, $c_{k,(k-1)} = P_k(1-P_k)^2(1-P_{k-1})(1-P_{k-2})$, and $K \geq k \geq 2$.

In order for the 1-hop link to be our bottleneck link,

$$\begin{aligned} U_{2,1}(s)\Big|_{s \in S(2,1)} &\geq U^*_{1,BS}(s)\Big|_{s \in S(1,BS)} \\ \Rightarrow \frac{4P_2(1-P_2)^2(1-q^*)}{K+K^2-2} &\geq \frac{2q^*(1-q^*)^3}{K+K^2} \\ \Rightarrow P_2(1-P_2)^2 &\geq q^*(1-q^*)^2 \frac{K+K^2-2}{2(K+K^2)}. \end{aligned} \tag{4}$$

Inequality (4) provides the condition on the link access probability of link $(2,1)$, which keeps $(1, BS)$ as the bottleneck link. For $K = 2$, the condition is violated when $P_2 > 0.7$ or $P_2 < 0.05$. For $K \to \infty$, the condition is violated when $P_2 > 0.7$ or $P_2 < 0.08$. Also, notice from Eqn. (4) that $c_{2,1}$ is maximized when $P_2 = 1/3$.

Similarly, for link $(3,2)$ and $K \geq 3$,

$$\begin{aligned} U_{3,2}(s)\Big|_{s \in S(3,2)} &\geq U^*_{1,BS}(s)\Big|_{s \in S(1,BS)} \\ \Rightarrow P_3(1-P_3)^2(1-P_2) &\geq q^*(1-q^*)^2 \frac{K+K^2-6}{3(K+K^2)} \end{aligned} \tag{5}$$

When the capacity of the link $(2,1)$ is maximized by setting $P^*_2 = 1/3$, Eqn. (4) is satisfied. This means that $0.05 \leq P_3 \leq 0.7$ satisfies the condition. In fact, as $P_2 \to 0.05$, we see that $(1-P_2)$ becomes large, and so dividing the R.H.S. of Eqn. (4) by this term would lower the bound even further. The special case occurs when $K \to \infty$ and $P_2 \to 0.7$. When $K \to \infty$ and $P_2 \to 0.7$, one can always assign $q^* = 0.25 < P_3 < 1/3$ to satisfy Eqn. (5).

For the link, $(k, k-1)$ and $K \geq k \geq 4$,

$$\begin{aligned} U_{k,k-1}(s)\Big|_{s \in S(k,k-1)} &\geq U^*_{1,BS}(s)\Big|_{s \in S(1,BS)} \\ \Rightarrow P_k\left(1-P_k\right)^2\left(1-P_{k-1}\right)\left(1-P_{k-2}\right) & \\ \geq P^*_1\left(1-P^*_1\right)^3 \frac{K^2+K-k(k-1)}{k(K^2+K)} & \end{aligned} \tag{6}$$

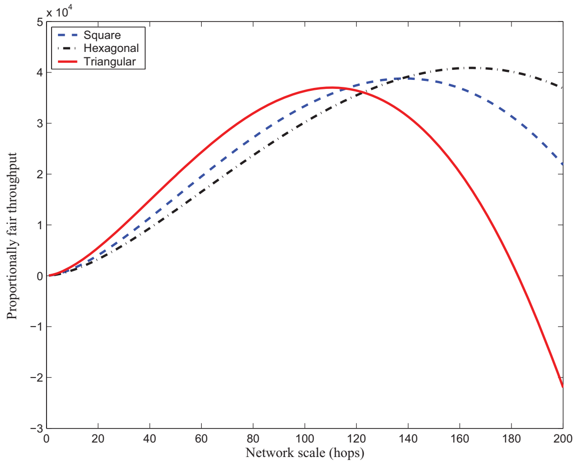

Fig. 1. Asymptotic behavior of proportionally fair throughput of each topology.

Here, we only show the weak dependency to P_k for the most restricting case, $K \to \infty$ and $k \to 4$, since it is easy to see that $(1 - P_1^*)\,\frac{K^2+K-k(k-1)}{k(K^2+K)}$ would remain reasonably small for most of the cases.

For $K \to \infty$ and $k \to 4$, Eqn. (6) becomes,

$$P_k\,(1-P_k)^2\,(1-P_{k-1})\,(1-P_{k-2}) \geq P_1^*\,(1-P_1^*)^2\,\frac{1}{6}.$$

This means that if $(1-P_{k-1})\,(1-P_{k-2}) < \frac{1}{6}$, then simply assigning $q^* = 0.25 < P_3 < 1/3$ would make the condition represented by Eqn. (6) true. This finally proves Lemma 4.2. ■

Lemma 4.2 is also true for the other two regular topologies. The proofs are similar and are omitted from this paper. However, we do note that the value of the optimal 1-hop probabilities, $p_{i,BS}^*$, of every regular topology, represents a feasible value for the transmission probability of every other node in the corresponding topology. This always guarantees that Lemma 4.2 holds for the other topologies as well.

By combining Lemmas 4.1 and 4.2, we prove the following theorem.

Theorem 4.3: In a regular topology, $y_s^* = \frac{c_{i,BS}}{|S(i,BS)|}$, $\forall s \in S$, $i \in N^{BS}$.

Theorem 4.3 indicates that if n and $|N^{BS}|$ is known, one can determine the optimal individual session rates that maximizes the total proportionally fair throughput. We provide the closed form expressions derived from this result, for all three regular topologies, in Table I.

A. Numerical Analysis

Theorem 4.3 enables us to compute the optimal session rate for all three regular topologies at any size, and obtain the closed form expressions for the total proportionally fair session rate as listed in Table I. Notice that the total proportionally fair session rate , $\sum_{\forall s \in S} \log y_s^*$, shown in Table I, consist of two product terms: $(K + K^2)$ and $\log(\bullet)$. For small values of K, the first product term is the dominating term in the expressions. However, as K becomes large, the logarithmic term becomes small and approaches a negative value. These two opposing terms will eventually create a turning point at some K, where the total proportionally fair session rate is maximized. To gain a deeper understanding of proportional fairness in regular topologies, we substitute a feasible optimal session rate with respect to a transmission rate of 250 kbps, which is used in WSNs with TELOSB motes, and vary the hop value, K, from 1 to 200 as shown as Fig. 1.

Fig. 1 reveals two very interesting characteristics of regular topologies, when the session rate is set to the optimal. First, for small scale regular topologies with each node sending at the optimal session rate, the triangular topology is better than the square topology, which in turn, is better than the hexagonal topology, in the context of total proportionally fair session rate optimization. However, this preference switches over for larger networks ($K > 120$ (hops) in this analysis), and the hexagonal topology is the most preferable for total proportionally fair session rate optimization. Second, for all three regular topologies, there exists an optimum size of the network, which maximizes their total proportionally fair session rate. This optimal size depends on the physical layer since y_s^* is just a ratio unless it is appropriately scaled by the transmission rate. Once the network's transmission rate is known, one can easily determine this optimum size K^*, for all three regular topologies. Often this optimum size may not be feasible given the current state of network technology. However, the potential for its use in large scale networks is growing, as seen in upcoming smart dust network.

V. Implementation

In order to provide experimental validation of our theoretical results, we deploy 12 TELOSB motes in an office environment, as a 2-hop square topology. Every mote is placed 2.5 inches above the ground, and the distance between every pair of motes is set to 2 feet. The transmission power of every mote is set to level 2, which transmits approximately upto 3 feet distances. The BS or sink node is placed in the center of the network. From preliminary experiments, we noticed that the BS overhears about 10% of the total packets transmitted from the mote that is 4 feet away. The MAC layer protocol of the motes is slotted ALOHA, which is implemented based on the MLA architecture developed in [9]. The slotted ALOHA MAC divides the transmission time into frames of length 2 milliseconds, and each frame contains two slots, each of length 1 millisecond. The first slot is used for time synchronization, and the second slot is used for packet transmission. In our experiments, the packet size is 32 bytes, including packet headers. This ensures that the packet transmission time is approximately equal to the slot length, given that the TELOSB antenna's data transmission rate is 250 Kbps [10]. Packets that are dropped due to collisions or buffer overflows are retransmitted until successful receipt at the next hop node. Each experiment runs from the time $T = 0$ to time , say T', the time it takes for 1000 packets from each of the 12 nodes to be received at the BS. The individual throughputs of the

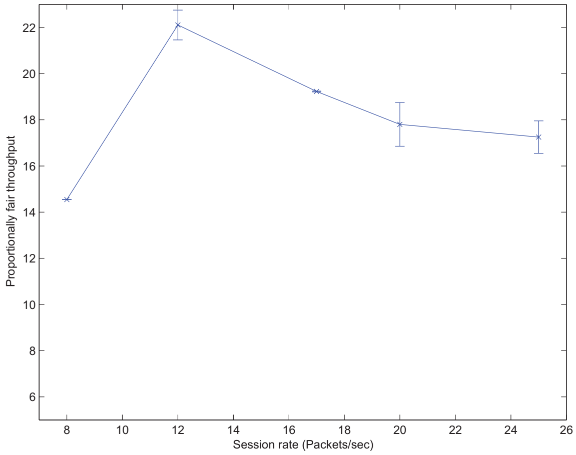

Fig. 2. Proportionally fair throughput at different session rates.

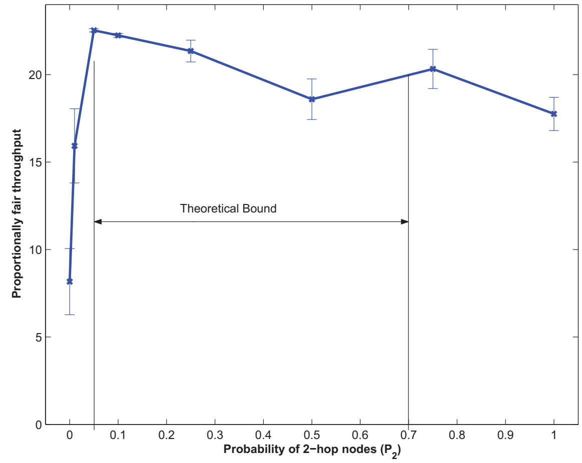

Fig. 3. Proportionally fair throughput when the channel access probability, P_2, at second hop varies from 0.01 to 1.

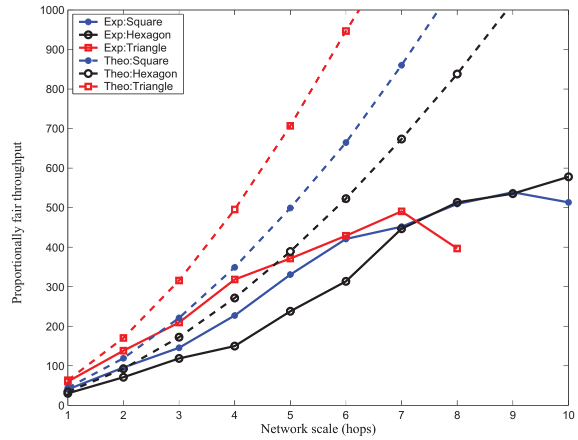

Fig. 4. Proportionally fair throughput of each topology at scales.

nodes are then calculated as $(1000/T')$ packets/sec, and used to calculate the total proportionally fair throughput for that experiment. The results of our experiments are calculated as an average of several runs (standard deviation is shown in Fig. 2 and Fig. 3), during which the orientation of the TELOSB motes are varied in order to minimize the error caused by the imperfect omnidirectional antennae. Through our experiments, we aim to determine the optimum session rate that maximizes the proportionally fair throughput of the network. We then study the effects of varying the link attempt probabilities of the 1-hop and 2-hop nodes, on the proportionally fair throughput.

A. Session Rate vs Proportionally Fair Throughput

In our choice of the domain of session rate values for the experiments, we are guided by our theoretical model. The optimal theoretical session rate, is 17.5 packets/second. In the real wireless medium, we expect the optimal rate to be higher than the theoretical value due to capture of colliding packets. As we vary the transmission rate from 8 (packets/sec) through 25 (packets/sec) (Fig. 2), we notice that the total proportionally fair throughput shows a sharp increase with increasing transmission rate and achieves its maximum at 12 (packets/sec). Additional increase in the session rate, causes the total proportionally fair throughput at the BS to decrease gradually as the link capacity at the BS approaches its saturation.

Unlike the ideal theoretical model which assumes the disk model for interference, in the real 2-hop square network, nodes that are not connected by a side of the square, but by the diagonals of the square, may be within the interference ranges of each other. This additional interference masks the positive effect of capture [11] on total log throughput and causes the experimental optimal rate to remain lower than our theoretical optimal solution.

B. Link Attempt Probability vs Fair Throughput

In order to verify our theoretical optimal 1-hop probabilities, we vary the 1-hop nodes' transmission probabilities from 0.125 to 1 and measure the total proportionally fair throughput for each run. Here, the transmission rate is set to the optimal value of 12 packets/second as determined in section IV A. We maintain the transmission probabilities of the 2-hop nodes to be $1/3$. Our results indicate that the maximum total proportionally fair throughput is achieved at 1-hop probability of $1/4$. As we vary the transmission probabilities of the nodes that are 2 hops from the base station while maximizing the 1-hop link capacity by setting $P_1 = 1/4$, we see from Fig. 3, that the total proportionally fair throughput remains high in the approximate range of $[0.05, 0.75]$ and tends to drop significantly as the 2-hop probability falls outside this range. This closely resembles the theoretical bound proposed in section IV.

VI. Simulation

The experimental implementation to observe the proportionally fair throughput becomes infeasible when we increase the size of the topology. In this section we compare the behaviour of each topology with different sizes using simulations. The simulations have been conducted using QualNet Simulator [12] which is a commercially available network simulation tool.

A. Simulation setup

We have implemented the slotted Aloha MAC protocol by modifying Qualnet's standard Aloha MAC protocol. We set the retransmission probabilities based on the topology used in accordance with Table I. At the physical layer, each node was configured to use 802.11b, transmitting at a data rate of 2 Mbps with an omni directional antenna model. The nodes generate constant bit rate traffic with a packet size of 500 bytes. Our theoretical model does not take into account the various delays that the packet suffers from the application layer to the physical layer, these delays include processing delay, queuing delay and network configuration delay. In order to compensate for this we set the session rate to a lower value than the calculated theoretical optimum session rate y_s^* for every node in every topology. We used the Bellman Ford routing protocol at the network layer, and from our observations we find that it routes the traffic in the network in a distributed manner. The simulations for each setting was run for 1000 seconds (which totals to 5000 packets sent for the higher session rates and 300 packets sent for the lower rates). The size of the topologies was varied from 1 hop to around 10 hops to observe the change in the proportionally fair throughput.

B. Simulation results

We see in Fig. 4 that for smaller network sizes, the triangular topology's proportionally fair throughput is higher than that of the square, which does better than the hexagonal topology. We also see the region where a crossover happens where the hexagonal topology does better than the square, which in turn, does better than the triangular topology, as predicted in our theoretical model. The simulations vary from the theoretical results in three aspects. The first is that the values of proportionally fair throughput in the simulations is lower than in theory. Secondly, at larger network sizes the proportionally fair throughput does not increase as sharply as in theory, and finally the simulations show the start of the crossover region, which in theory was expected to occur at a much larger network size of around two hundred hops. This variation between the simulations and theory could be due to a number of contributing factors. The first is that in our simulation the proportionally fair throughput is calculated for each node, by summing the log of the received throughput at the BS, while in theory it is calculated as the sum of the log of the session rates of all the sessions. This is a much higher value than the actual received rate at the BS, which could explain why the simulations have lower values. The second factor is that in the simulations as we move farther away from the BS, there is a higher probability that the nodes packet will not reach the BS, due to collision or dropped packets along the path. We observed that as the network grew in size the number of nodes that could not send any packets to the sink increased. Our theoretical model does not take into account this more realistic collision model which could contribute to the difference in the results.

VII. Conclusion

In this paper, we address the problem of end-to-end proportionally fair rate allocation in regular lattice topologies of WSNs. We derive the optimal session rates and channel access probabilities of every node, in each regular topology, that would achieve the maximum proportionally fair throughput. Our solution is counter-intuitive and reveals special characteristics unique to regular node deployments, which can be exploited during network design. We also perform experiments using TELOSB sensors in order to assess how our results would impact a WSN in a real-world environment, and find that our solution optimizes the fair throughput of the network. Our finding from experiments and simulations indicates that possible future work could include enhancing our theoretical model with a better representation of the physical layer. This would include using a more realistic path loss model, capture model and multi packet reception model.

References

[1] Y. Gao, K. Wu, and F. Li, "Analysis on the redundancy of wireless sensor networks," in *WSNA '03: Proceedings of the 2nd ACM international conference on Wireless sensor networks and applications*. New York, NY, USA: ACM, 2003, pp. 108–114.

[2] Y. Wang and D. P. Agrawal, "Optimizing sensor networks for autonomous unmanned ground vehicles," E. M. Carapezza, Ed., vol. 7112, no. 1. SPIE, 2008, p. 711215. [Online]. Available: http://link.aip.org/link/?PSI/7112/711215/1

[3] G. Mergen and L. Tong, "Stability and capacity of regular wireless networks," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 1938–1953, 2005.

[4] R. Rajagopalan and P. Varshney, "Connectivity analysis of wireless sensor networks with regular topologies in the presence of channel fading," *IEEE Transactions on Wireless Communications*, vol. 8, no. 7, pp. 3475–3483, 2009.

[5] L. E. Li, M. Pal, and Y. R. Yang, "Proportional fairness in multi-rate wireless LANs," in *Proceedings of IEEE INFOCOM*, Phoenix, AZ, Apr. 2008.

[6] X. Wang and K. Kar, "Cross-layer rate control for end-to-end proportional fairness in wireless networks with random access," in *MobiHoc '05: Proceedings of the 6th ACM international symposium on Mobile ad hoc networking and computing*. New York, NY, USA: ACM, 2005, pp. 157–168.

[7] H. Tian, H. Shen, and M. Roughan, "Maximizing networking lifetime in wireless sensor networks with regular topologies," in *PDCAT '08: Proceedings of the 2008 Ninth International Conference on Parallel and Distributed Computing, Applications and Technologies*. Washington, DC, USA: IEEE Computer Society, 2008, pp. 211–217.

[8] X. Liu and M. Haenggi, "The impact of the topology on the throughput of interference-limited sensor networks with Rayleigh fading," in *Sensor and Ad Hoc Communications and Networks, 2005. IEEE SECON 2005. 2005 Second Annual IEEE Communications Society Conference on*, 2005, pp. 317–327.

[9] K. Klues, G. Hackmann, O. Chipara, and C. Lu, "A component based architecture for power-efficient media access control in wireless sensor networks," in *Proceedings of 5th ACM Conference on Embedded Networked Sensor Systems (SenSys 2007)*, November 2007. [Online]. Available: http://www.cse.wustl.edu/ lu/papers/sensys07.pdf

[10] M. Inc, "Telosb Data sheet," *Crossbow Inc., SanJose, California*.

[11] J. H. Jun, A. Syed, and B. Krishnamachari, "The impact of capture on multihop wireless networks in an optimal rate control framework," in *Proceedings of the 3rd international conference on Wireless internet*. ICST (Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering), 2007, pp. 1–10.

[12] Qualnet network simulator, scalable network technologies, inc. [Online]. Available: www.qualnet.com