Capacity of Multi-hop Wireless Network with Frequency Agile Software Defined Radio

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Abstract—**With the advance of both hardware and software technologies, the concept of software defined radio (SDR) is becoming more and more popular in the academic and industrial communities. Its popularity has been increased by the recent intensive research of cognitive radio technology, which is built on top of SDR. One of key features of SDR is its capability of frequency agility, which means a single SDR can access multiple channels, subject to a certain total frequency bandwidth (channel span) constraint. Compared with the traditional multiple-radio solutions, the SDR setup has the advantages of higher flexibility and reduced hardware size (cost). In this paper, we investigate the achievable capacity of a wireless network with single-SDR equipped transceivers, especially a multiple-hop network, for any given network flows. We propose new approaches to formulate the single-SDR constraint, which is unique for the derivation of capacity upper bounds. We also propose a heuristic algorithm to obtain a lower bound for the capacity. Both numerical and simulation results are presented to demonstrate the potential capacity for an SDR network and compared it with a multiple-radio network.**

I. Introduction

To meet the increasing demand for high-rate wireless communication services, utilizing multiple channels for simultaneous transmissions is one effective way. Recent development of dynamic spectrum management has offered more opportunistic spectrum resources for data communication networks. It is expected that there will be more and more wireless networks using wider spectrum bands and more spectrum channels in the near future.

For a wireless multi-hop network, to utilize multiple channels for performance enhancement, the traditional way is to use multiple radios at each node, the so called multi-radio (MR) multi-channel networks. For IEEE 802.11 mesh networks operating at 2.4 GHz and 5 GHz ISM bands, there are a total number of 15 orthogonal channels, which may not be overwhelming for the MR configuration. However, with an increasing number of usable channels, such as in a dynamic spectrum access scenario, where secondary users can opportunistically use channels from a large spectrum pool offered by primary users, the performance of such an MR approach is limited by the number of radios equipped by a node (due to both size and cost concerns). A concrete example is the recent FCC rule [1] to allow unlicensed use of TV bands, which can offer a large number of channels (any TV channel within channel 7 to channel 51 in North America, each with 6 MHz). Such a large number of potentially available channels definitely imposes a significant challenge for networks with the MR configuration.

Software defined radio (SDR) is an advanced radio technology, which can dynamically change its transmission parameters to achieve high performance. One capability is *frequency agility*, i.e., a single SDR is able to use multiple channels (possibly from discontiguous spectrum segments) simultaneously for data transmission. Such a feature can be realized by some well developed technologies such as orthogonal frequency-division multiplexing (OFDM) [2]. In the case of a dynamic spectrum access network, such frequency agility helps secondary users to exploit spectrum holes more effectively: primary users' channel access leaves discontiguous available spectrum segments for secondary users' opportunistic usage. However, such frequency agility also has some practical limitations such as spectrum bandwidth (channel) span, which is considered in this work. For example, USRP2 hardware can support 25MHz frequency bandwidth at most [3], while WhiteFi only supports 20MHz at most [4].

The capacity of multi-radio multi-channel wireless mesh networks is well studied, such as the two notable works in [5][6]. There are several existing studies in resource allocation and protocol design for SDR networks. The uniqueness of uneven channel sub-division and spectrum heterogeneity in an SDR mesh network is considered in [7]. The end-to-end bandwidth allocation in mesh networks with cognitive radios is studied in [8], which involves routing, scheduling, and spectrum allocation. These works differ from ours in that they all assume multiple radios in the system model. In [9], Choi et al. propose a protocol to allocate continuous channels to each node so as to maximize the network throughput. The spectrum span due to the hardware constraint is also considered in their problem. They evaluate the protocol with both simulations and experiments, without theoretical performance bounds.

Different from the existing studies, in this work, we study the capacity (achievable throughput) of a multi-hop wireless network with a single SDR at each node for any given network flows, while taking the SDR limitation into consideration. Due to the potential application of SDR networks in dynamic spectrum access, we also incorporate the spectrum availability into our study. Different from MR networks where the constraint on the number of radios can be easily expressed in a linear format, the single-SDR constraint is not straightforward to characterize. To address this issue, we propose an approach to add a new auxiliary variable, called *virtual radio*, for the single-SDR constraint. With the help of this variable, we are able to formulate the joint link scheduling, channel allocation, and flow routing in a linear format. Considering the high complexity of this optimization problem, we apply relaxation approaches based on necessary conditions, which leads to an upper bound of the network capacity using linear optimization tools. Based on the upper bound results, we further propose a heuristic algorithm to obtain a lower bound for the capacity. Specifically, the upper bound results provide ratios for the link flow assignment. In our heuristic, we use a greedy approach to achieve such a link flow assignment, which deals with all the constraints.

The rest of this paper is organized as follows. In Section

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II, we describe the system model under consideration. Section III formulates the optimization problem. We propose a linear relaxation approach to obtain the upper bound capacity in Section IV. Section V presents the heuristic algorithm for the lower bound capacity. Performance evaluation based on numerical and simulation results is presented in Section VI. Section VII draws conclusions of this research.

II. System Model

We consider a multi-hop wireless network with a set N of static nodes. The network can be a secondary network in that it can opportunistically use channels from primary networks. The network supports a set of data flows F, which is characterized by a set of source nodes $S = \{s(f) : f \in F\}$, a set of destination nodes $D = \{d(f) : f \in F\}$, and the associated end-to-end rate demand $R = \{r(f) : f \in F\}$.

A. Channel availability

There is a set K of orthogonal channels in the system, indexed from the lowest frequency to the highest frequency by $1, 2, \ldots, |K|$. All the channels have the same frequency bandwidth, and form a continuous spectrum band. Each channel can be occupied by one or several primary users. The primary users have synchronized and time-slotted channel access. The usable channels of secondary nodes are subject to primary users' channel usage. We focus on the problem for a single time period P with fixed length L. At the beginning of this period, spectrum availability information is obtained either by spectrum sensing components of secondary nodes or via an external spectrum database. In either case, it is assumed that the spectrum availability information is accurate, and will be collected by a central node for the computation of resource allocation. The resource allocation results will be sent to secondary nodes for data transmission in the period P.

Let a_i^k denote the availability of channel k at node i for the period P, with $a_i^k = 1$ if usable and $a_i^k = 0$ otherwise. The channel availability does not change during the period P. The time period P is divided into time slots with equal length τ, with a total number of T time slots. While the total length of the time period $L = T\tau$ is fixed, both τ and T can be changed.

B. Communication links

Two nodes can form a communication link if and only if both nodes have common available channels and they are within each other's communication range. The communication ranges of all nodes in all the channels are the same (symmetric links), denoted by R_C. Similarly, the interference range is denoted by R_I, with $R_I \geq R_C$. Let E denote the set of potential communication links, i.e., link (i, j) between nodes i and j is in E if and only if $dist(i, j) < R_C$, where $dist(i, j)$ is the distance between the two nodes. Similarly, let I denote the set of interference relations between any two nodes in the network: link (i, j) is in I if and only if $dist(i, j) < R_I$. The neighbour set of node i is denoted by $M_i = \{j | dist(i, j) < R_C\}$. Each node operates in a half duplex mode, i.e., it cannot transmit and receive simultaneously.

If a link uses a channel for transmission, it can achieve a certain data rate determined by the transmission power and channel condition. Since the interference-free channel allocation is considered, the effect of other transmissions is neglected. We use C_{ij}^k to denote the data rate from node i to node j using channel k, which is a constant.

C. Scheduling

The secondary network is time-slotted with synchronization, over the time period P. We use $y_{ij}(t)$ to denote the scheduling at $t \in [1, T]$: $y_{ij}(t) = 1$ if node i transmits to node j in time slot t; and $y_{ij}(t) = 0$ otherwise. Due to the assumption of a single half-duplex radio for each node, we have the following scheduling constraint that a node can either transmit to or receive from at most one of its neighbours in any time slot, but not both, given by

$$\sum_{j \in M_i} y_{ij}(t) + y_{ji}(t) \leq 1, \quad \forall i \in N. \tag{1}$$

D. Channel allocation

An active link can use several channels for transmission. A channel allocation is denoted by $x_{ij}^k(t)$: $x_{ij}^k(t) = 1$ if link (i, j) transmits data from node i to node j using channel k in time slot t; and $x_{ij}^k(t) = 0$ otherwise. A feasible channel allocation is subject to a particular scheduling

$$x_{ij}^k(t) = 0, \text{ if } y_{ij}(t) = 0, \quad \forall (i, j) \in E, \forall k \in K.$$

Such a constraint can be expressed in the linear form as

$$x_{ij}^k(t) \leq y_{ij}(t), \quad \forall (i, j) \in E, \forall k \in K. \tag{2}$$

Channel allocation is also constrained by channel availability, given by

$$x_{ij}^k(t) \leq a_i^k \cdot a_j^k, \quad \forall (i, j) \in E, \forall k \in K. \tag{3}$$

Furthermore, for each channel, active links must satisfy an interference constraint. This constraint requires that no active links are within each other's interference range in each channel. Therefore, for each interference pair $(i, j) \in I$, at most one link incident on either node i or j is active

$$\sum_{(i,i'):i' \in M_i} x_{ii'}^k(t) + \sum_{(i',i):i \in M_{i'}} x_{i'i}^k(t) + \sum_{(j,j'):j' \in M_j} x_{jj'}^k(t) + \sum_{(j',j):j \in M_{j'}} x_{j'j}^k(t) \leq 1, \quad \forall (i, j) \in I, \forall k \in K. \tag{4}$$

E. SDR constraint

Each node has a single SDR. The radio can span at most $\kappa \leq |K|$ channels, and has the capability to transmit on discontiguous channels. For the channels used by an active link, denote the channel with the lowest index as channel c_l, and the channel with the highest index as channel c_h. Then we have $x_{ij}^{c_l}(t) = 1$ and $x_{ij}^{c_h}(t) = 1$, while channel k ($c_l < k < c_h$) can be either used or not. The channel span limit requires that $c_h - c_l + 1 \leq \kappa$, which is a combinatorial constraint.

Such a constraint can be described using *channel bundles*. Here channel set K contributes to $(|K| - \kappa + 1)$ channel bundles, denoted by channel bundle set B. The channel bundle with index q consists of channel set $B_q = \{q, q+1, \ldots, q+\kappa-1\}$. Let $b_{ij}^q(t)$ denote the channel bundle allocation, similar to $x_{ij}^k(t)$. The SDR constraint requires that at most one channel bundle is used by a radio at any slot

$$\sum_{j \in M_i} \sum_{q \in B} (b_{ij}^q(t) + b_{ji}^q(t)) \leq 1, \quad \forall i \in N. \tag{5}$$

The channel bundle used by a radio is also constrained by scheduling

$$b_{ij}^q(t) \le y_{ij}(t), \quad \forall (i,j) \in E, \forall q \in B \tag{6}$$

which can replace constraint (2).

As a result, the channel allocation is subject to this additional constraint, written as

$$x_{ij}^k(t) = 0, \text{ if } k \notin \bigcup_{b_{ij}^q(t)=1} B_q \tag{7}$$

which is difficult to be expressed in a linear format.

F. Flow routing

For the multi-hop data forwarding, multi-path routing is used, instead of single path routing. We use g_{ij}^f to denote the average data rate on link (i,j) for flow f over the time period P (from node i to j),

$$g_{ij}^f = \frac{1}{T} \sum_{t=1}^{T} g_{ij}^f(t), \quad \forall f \in F$$

where $g_{ij}^f(t)$ is the data rate on link (i,j) for flow f in slot t.

The traffic flow should satisfy flow constraints. For each of the relay nodes, the incoming flow should be equal to the outgoing flow, which is the flow reservation constraint, given by

$$\sum_{j \in M_i} g_{ij}^f = \sum_{j \in M_i} g_{ji}^f, \quad \forall i \ne s(f), i \ne d(f), \forall f \in F. \tag{8}$$

We want to maximize the value λ, such that a data rate $\lambda \cdot r(f)$ can be achieved for flow f. Thus for each of the source nodes, we have

$$\sum_{j \in M_i} g_{ij}^f = \lambda \cdot r(f), \quad \forall i = s(f), \forall f \in F. \tag{9}$$

Similarly for destination nodes,

$$\sum_{j \in M_i} g_{ji}^f = \lambda \cdot r(f), \quad \forall i = d(f), \forall f \in F. \tag{10}$$

The aggregated data rate on a link cannot exceed its capacity,

$$\sum_{f \in F} g_{ij}^f \le \frac{1}{T} \sum_{t=1}^{T} \sum_{k \in K} x_{ij}^k(t) C_{ij}^k, \quad \forall (i,j) \in E. \tag{11}$$

III. Problem Statement

Our objective is to maximize λ, where at least $\lambda \cdot r(f)$ data rate can be routed for each flow f. We need to determine the following variables

1) a transmission rate over link (i,j) for flow f: g_{ij}^f
2) a channel bundle allocation $b_{ij}^q(t)$
3) a channel allocation $x_{ij}^k(t)$
4) a scheduling scheme $y_{ij}(t)$
5) a rate scaling factor λ.

The optimization goal for this problem (P1) is

$$\begin{aligned} \max \quad & \lambda \\ s.t. \quad & (1)(2)(3)(4)(5)(6)(7)(8)(9)(10)(11) \end{aligned} \tag{P1}$$

where the main difference from the optimization in a traditional mesh network is in constraint (7) due to the spectrum span limit of SDR. Such uniqueness makes the problem more difficult.

IV. Linear Relaxation

One typical way to solve the above joint channel assignment, scheduling, and routing problem is to apply linear relaxation first to reduce the complexity. Following is an attempt to make the problem linear.

The original problem needs to determine the value of channel allocation $x_{ij}^k(t)$ and scheduling $y_{ij}(t)$ for each time slot $t \in [1,T]$, which is with extremely high complexity. Therefore, we first reduce such complexity by averaging over T time slots. The average data rate on channel k over link (i,j), for a given channel allocation, is defined as

$$h_{ij}^k = \frac{1}{T} \sum_{i=1}^{T} x_{ij}^k(t) C_{ij}^k. \tag{12}$$

We define a variable associated with $y_{ij}(t)$, the average activation of link (i,j),

$$H_{ij} = \frac{1}{T} \sum_{t=1}^{T} y_{ij}(t).$$

As a result, the constraints in (1)(2)(3)(4)(11) can be changed by summing over T slots, and using the above two definitions,

$$\sum_{j \in M_i} H_{ij} + H_{ji} \le 1, \quad \forall i \in N \tag{13}$$

$$\frac{h_{ij}^k}{C_{ij}^k} \le H_{ij}, \quad \forall (i,j) \in E, \forall k \in K \tag{14}$$

$$\frac{h_{ij}^k}{C_{ij}^k} \le a_i^k \cdot a_j^k, \quad \forall (i,j) \in E, \forall k \in K \tag{15}$$

$$\begin{aligned} &\sum_{(i,i'):i' \in M_i} \frac{h_{ii'}^k}{C_{ii'}^k} + \sum_{(i',i):i \in M_{i'}} \frac{h_{i'i}^k}{C_{i'i}^k} + \sum_{(j,j'):j' \in M_j} \frac{h_{jj'}^k}{C_{jj'}^k} \\ &+ \sum_{(j',j):j \in M_{j'}} \frac{h_{j'j}^k}{C_{j'j}^k} \le 1, \quad \forall (i,j) \in I, \forall k \in K \end{aligned} \tag{16}$$

$$\sum_{f \in F} g_{ij}^f \le \sum_{k \in K} h_{ij}^k, \quad \forall (i,j) \in E. \tag{17}$$

The difficulty comes from the SDR constraint (7). It is not straightforward how to achieve linear relaxation. In the following, we propose an approach to deal with such a challenge.

A. Virtual radios

We transform the original problem by creating virtual radios so that the explicit SDR constraint can be incorporated into virtual radios. The detailed transformation procedure is described as follows.

For the original network, we replace each node's single SDR radio with a set V of frequency-fixed radios, $|V| = |K| - \kappa + 1$, which are referred to as *virtual radios*. Each virtual radio can span κ continuous channels, but is fixed at these channels (a channel bundle). Virtual radio q of node i corresponds to channel bundle q, which is able to use channels in channel bundle q. Virtual radio q of a node can only communicate with virtual radio q in one of its neighbouring nodes. Define variable $v_{ij}^q(t)$: $v_{ij}^q(t) = 1$ if virtual radios q of both nodes i and j are used on link (i,j) at time slot t; and $v_{ij}^q(t) = 0$ otherwise. At any time slot, only a single virtual radio based link can be active for any node, i.e.,

$$\sum_{j \in M_i} \sum_{q \in V} (v_{ij}^q(t) + v_{ji}^q(t)) \leq 1, \quad \forall i \in N. \tag{18}$$

Note that in channel assignment for each virtual radio, we define variable $z_{ij}^{qk}(t)$: $z_{ij}^{qk}(t) = 1$ if link (i,j) uses virtual radio q to communicate on channel k, where $k \in B_q$. The scheduling constraint can be represented by

$$z_{ij}^{qk}(t) \leq v_{ij}^q(t), \quad \forall (i,j) \in E, \forall q \in V, \forall k \in B_q \tag{19}$$

and the channel availability constraint by

$$z_{ij}^{qk}(t) \leq a_i^k \cdot a_j^k, \quad \forall (i,j) \in E, \forall q \in V, \forall k \in B_q. \tag{20}$$

The corresponding interference constraint is

$$\sum_{(i,i'):i' \in M_i} \sum_{q:k \in B_q} z_{ii'}^{qk}(t) + \sum_{(i',i):i \in M_{i'}} \sum_{q:k \in B_q} z_{i'i}^{qk}(t) + \sum_{(j,j'):j' \in M_j} \sum_{q:k \in B_q} z_{jj'}^{qk}(t) + \sum_{(j',j):j \in M_{j'}} \sum_{q:k \in B_q} z_{j'j}^{qk}(t) \leq 1, \quad \forall (i,j) \in I, \forall k \in K. \tag{21}$$

The main difference between the virtual radios and multiple radios is that interfering links using virtual radios with overlapping channel bundles can be active simultaneously, as long as the channels actually used by these active virtual radios are all different.

The link capacity constraint is

$$\sum_{f \in F} g_{ij}^f \leq \frac{1}{T} \sum_{t=1}^{T} \sum_{q \in V} \sum_{k \in B_q} z_{ij}^{qk}(t) C_{ij}^k, \quad \forall (i,j) \in E. \tag{22}$$

Therefore, all the constraints are expressed in a linear format now. Similar to using variables h_{ij}^k and H_{ij}, we can define variables l_{ij}^{qk} and L_{ij}^q to relax the constraints (18)-(22)

$$l_{ij}^{qk} = \frac{1}{T} \sum_{t=1}^{T} z_{ij}^{qk}(t) C_{ij}^k, \quad L_{ij}^q = \frac{1}{T} \sum_{t=1}^{T} v_{ij}^q(t).$$

Finally, we are able to make the original problem in a linear programming format. The new linear programming problem (P2) is given by

$$\max \quad \lambda$$

$$s.t.$$

$$\sum_{j \in M_i} \sum_{q \in V} (L_{ij}^q + L_{ji}^q) \leq 1, \quad \forall i \in N$$

$$\frac{l_{ij}^{qk}}{C_{ij}^k} \leq L_{ij}^q, \quad \forall (i,j) \in E, \forall q \in V, \forall k \in B_q$$

$$\frac{l_{ij}^{qk}}{C_{ij}^k} \leq a_i^k \cdot a_j^k, \forall (i,j) \in E, \ \forall q \in V, \forall k \in B_q$$

$$\sum_{(i,i'):i' \in M_i} \sum_{q:k \in B_q} \frac{l_{ii'}^{qk}}{C_{ii'}^k} + \sum_{(i',i):i \in M_{i'}} \sum_{q:k \in B_q} \frac{l_{i'i}^{qk}}{C_{i'i}^k} + \sum_{(j,j'):j' \in M_j} \sum_{q:k \in B_q} \frac{l_{jj'}^{qk}}{C_{jj'}^k} + \sum_{(j',j):j \in M_{j'}} \sum_{q:k \in B_q} \frac{l_{j'j}^{qk}}{C_{j'j}^k} \leq 1, \quad \forall (i,j) \in I, \forall k \in K$$

$$\sum_{f \in F} g_{ij}^f \leq \sum_{q \in V} \sum_{k \in B_q} l_{ij}^{qk}, \quad \forall (i,j) \in E$$

$$\sum_{j \in M_i} g_{ij}^f = \sum_{j \in M_i} g_{ji}^f, \forall i \neq s(f), i \neq d(f), \forall f \in F$$

$$\sum_{j \in M_i} g_{ij}^f = \lambda \cdot r(f), \quad \forall i = s(f), \forall f \in F$$

with respect to the optimization variables l_{ij}^{qk}, L_{ij}^q, g_{ij}^f, and λ.

The relaxation enlarges the optimization space. Hence, the solution to this linear programming problem corresponds to an upper bound of the solution to the original problem, which may not have a feasible solution.

V. Heuristic Algorithm

Once we obtain the solution for the linear programming problem P2, we can use the results to achieve a lower bound of the capacity based on a heuristic algorithm. Here we apply a heuristic algorithm based on Packing Dynamic Channel Assignment (PDCA) algorithm in [5]. The main idea is to pack the flows in a greedy manner in each time slot. While making the greedy link channel allocation, we need to make sure all the constraints are satisfied so that the allocation is feasible.

As in the original PDCA, we first aggregate all the flows on different channels on a given link into a single flow on the link. Denote the link flow variable in the solution of P2 as $\hat{g}_{ij}^f$, then the total required data rate on link (i,j) over time period P is the summation of all the flows, i.e., $\hat{g}_{ij} = \sum_{f \in F} \hat{g}_{ij}^f$. To find a lower bound of the capacity, we use $\hat{g}_{ij} L$ as the required amount of data for link (i,j) for the duration L of time period P. Meanwhile, we use an initial value for the time slot length, denoted by τ_o. Note that we need to set the value of τ_o such that $\hat{g}_{ij} L$ is much larger than $C_{ij}^k \tau_o$. Otherwise, in a case where the demand of the flows is less than the link capacity, the results of the heuristic algorithm will not give the network's real capacity, as many links are underutilized. Since $\hat{g}_{ij}$ is an upper bound, the required time duration must be longer than L. We will scale the resulted duration to L as shown later in this section.

In our heuristic algorithm, we record the remaining required amount of data on link (i,j) as u_{ij}. At the beginning of each time slot, we sort the links in the descending order of u_{ij}. For the first link (i,j) in the sorted list, we choose *feasible* channels for it, and decrease the value of u_{ij} accordingly. We then check the next link in the list whether it can be allocated with some channels in this slot, and allocated the channels if possible. Repeat the process for each link in the list one by one until the end of the list. If there are still unsatisfied links, we move on to the next slot and repeat the allocation process.

To ensure the feasibility during the channel assignment in each slot, we adhere to the constraints of scheduling, channel allocation, and SDR. While the first two constraints can be checked for each channel independently, the SDR constraint brings more complexity. Here, the heuristic method is based on the channel bundle. Specifically, for each channel bundle, we exclude infeasible channels according to the scheduling and channel allocation constraints, and calculate its rate as $C_{ij}^q = \sum_{k\in B_q,\text{ and feasible}} C_{ij}^k$. We then assign the feasible channels in channel bundle p for link (i, j), where channel bundle p has the maximum rate, i.e., $p = \arg\max_{q\in B} C_{ij}^q$. The time complexity for each slot is $O(|E|(|E| + |K|))$. Finally, the algorithm finishes with the number of time slots T needed for the required amount of data $g_{ij}L$. We obtain a lower bound for the rate scaling factor λ, given by $\frac{L}{T\tau_o}\hat{\lambda}$, where $\hat{\lambda}$ is from the solution of P2. However, the total length of T time slots, $T\tau_o$, will exceed the original time period length L, since we do not impose such a constraint at the beginning. To address this issue, we can simply shrink the length of a time slot from τ_o to $\tau = \frac{L}{T}$. As a result, the transmitted amount of data for link (i, j) in the time period P is $\frac{\tau}{\tau_o}\hat{g}_{ij}L$.

VI. Evaluation

We use simulations to evaluate performance of the SDR network, and compare the lower bound and upper bound capacities of the SDR network with the capacity of the corresponding MR network. Both grid and random network topologies are used in our simulations. We examine key parameters affecting the system performance, including the SDR channel span κ (for the MR network, κ is the number of independent radios), the channel number $|K|$, and primary users' traffic in terms of the probability of channel availability for secondary users $Pr(a_i^k = 1)$. In all the evaluations, we assume that the data rate of each communication link is one unit value, i.e., $C_{ij}^k = 1$. The interference range is the same as the communication range. We leverage the optimization tool *linprog* of MATLAB to solve the linear programming problems for both SDR and MR networks.

A. MR network

Before presenting simulation results, we first briefly describe the MR network, its optimization problems and solutions. An MR node can communicate with neighbouring nodes on κ channels simultaneously using its κ separate radios. Unlike the SDR node, the MR node can use any up to κ channels within K channels. We refer to such a constraint as the MR constraint. The optimization problem for the MR network is formulated as a mixed integer linear programming problem similar to the SDR network. Specifically, we define $x_{ij}^k(t)$ to indicate whether channel k is used for link (i, j) in time slot t. We have the MR constraint as

$$\sum_{j\in M_i}\sum_{k\in K} x_{ij}^k(t) + x_{ji}^k(t) \le \kappa, \quad \forall i \in N. \tag{23}$$

The optimization problem for the MR network (P3) is

$$\begin{aligned} \max \quad & \lambda \\ s.t. \quad & (3)(4)(8)(9)(10)(11)(23) \end{aligned} \tag{P3}$$

with respect to $x_{ij}^k(t)$, g_{ij}^f, and λ.

After applying the similar linear relaxation with the additional variable in (12), we can transfer (23) to

$$\sum_{j\in M_i} \frac{h_{ij}^k}{C_{ij}^k} + \frac{h_{ji}^k}{C_{ji}^k} \le \kappa, \quad \forall i \in N. \tag{24}$$

The relaxed problem for the MR network (P4) is then given by

$$\begin{aligned} \max \quad & \lambda \\ s.t. \quad & (15)(16)(8)(9)(10)(17)(24) \end{aligned} \tag{P4}$$

with respect to h_{ij}^k, g_{ij}^f, and λ. Solving P4 with the optimization tool leads to the solution of an upper bound. The heuristic algorithm (PDCA) is used to obtain a lower bound, using the upper bound as its input [5].

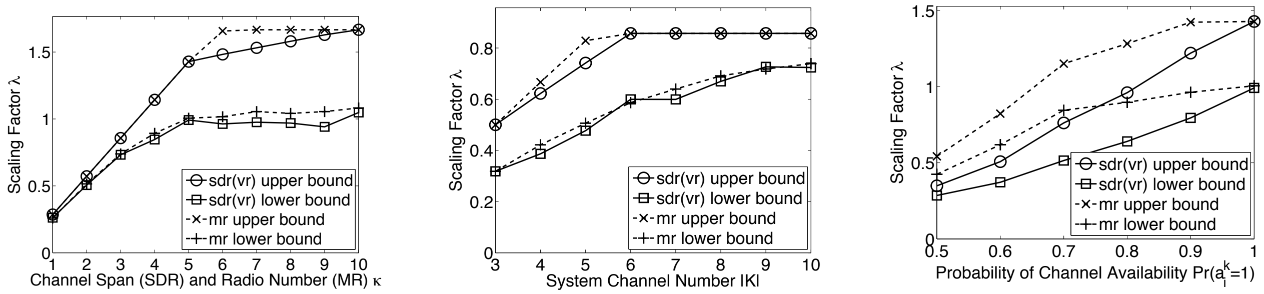
B. Grid Topology

For the grid topology, we have 25 secondary nodes arranged in a 5 by 5 grid. Each node has at most 4 neighbours. There are 10 flows, 5 horizontal flows, and 5 vertical flows in the network. Each flow requires a unit data rate.

Figure 1(a) shows the results when we change the channel span κ of SDR nodes. Here the total number of channels is fixed, and all the channels are always available. When both the channel span for the SDR network and the radio number for the MR network increase, the upper bound of the MR network capacity is always greater than the upper bound of the SDR network capacity. This is because a node in the MR network can access any κ channels, while a node in the SDR network can only access the ones in a channel bundle with κ continuous channels. The solution search space for the MR network is larger than the one for the SDR network. The lower bounds of the capacity for the two networks are close. The greedy allocation in the heuristic algorithms for both SDR and MR networks serves a link with a large flow demand before that with a small flow demand. Together with the setting that all the channels are always available, it results in a similar channel allocation for both SDR and MR networks. Only after links of a large demand are served, the small demand links will be taken care of in the last few slots of the time period. During these slots, the MR network can make more efficient utilization of the channel resources than the SDR network for each slot.

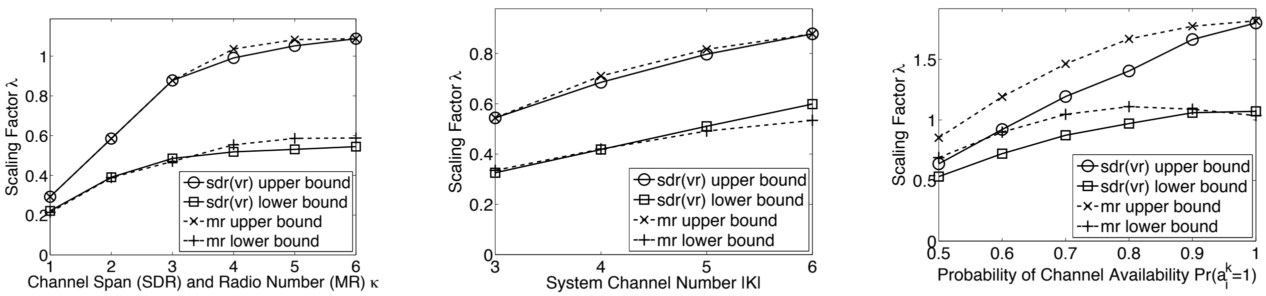
The effect of the total channel number $|K|$ is shown in Fig. 1(b), under the assumption that all the channels are always available. Given the channel span, capacities for both SDR and MR networks increase with the channel number, since more channel resources can be utilized. As the channel number further increases, the capacities become flattened, due to the limited channel span for SDR nodes and the limited radio number for MR nodes. As expected, both the upper bound and lower bound of the MR network capacity are higher than those of the SDR network capacity, similar to Fig. 1(a). However, the potential performance of the SDR network is comparable to that of the MR network, when all the channels are always available.

However, as the probability of channel availability decreases, the MR network outperforms the SDR network more significantly, as shown in Fig. 1(c). We randomly generate the channel availability at each node, according to the probability of a channel being available $Pr(a_i^k = 1)$. With several available channels scattering around in the channel set K, an MR node can make better usage of channels in the whole set K subject to the radio number constraint, while an SDR node can only select channels in a single channel bundle. With the same value of κ for both networks, the capacity of the MR configuration can be almost 2 times of that of the SDR configuration, at the cost of more radios.



(a) λ vs. κ with $Pr(a_i^k = 1) = 1$ (b) λ vs. $|K|$ with $Pr(a_i^k = 1) = 1$ (c) λ vs. $Pr(a_i^k = 1)$

Fig. 1: Grid topology with $|K| = 10$



(a) λ vs. κ with $Pr(a_i^k = 1) = 1$ (b) λ vs. $|K|$ with $Pr(a_i^k = 1) = 1$ (c) λ vs. $Pr(a_i^k = 1)$

Fig. 2: Random topology with $|K| = 6$

C. Random Topology

For the configuration of random network topologies, we have 25 secondary nodes randomly located in an area of 100 units by 100 units. The communication range and interference range are both 30 units. Each random topology is ensured to be a connected graph. There are 5 randomly generated flows. We generate multiple random graphs and associated flows, and take their average for all the results in this part. Each flow has a unit data rate.

The effects of the channel span and the channel number are presented in Fig. 2(a) and Fig. 2(b) with all the channels always available. The performance of the SDR network is very close to that of the MR network.

When the probability of channel availability decreases, the SDR network using the virtual radio scheme performs poorly when compared with the MR network, as shown in Fig. 2(c).

VII. Conclusion

In this paper, we study the capacity for a multi-hop wireless network of nodes equipped with a single SDR, which can simultaneously use multiple channels for data communication subject to the channel span constraint. We formulate the joint scheduling, channel allocation and routing problem for the optimal network capacity given any number of flows, considering the unique single-SDR constraint and the traditional MR network constraint. We propose a virtual radio scheme to simplify the calculation of the capacity. Based on this scheme, we are able to get a capacity upper bound of the SDR network. Using such an upper bound result, our proposed heuristic algorithm can obtain the lower bound of the capacity. Numerical results demonstrate the comparable performance of SDR and MR networks when the channels have a high availability probability. The MR network outperforms the SDR network more significantly, when the channels are with a moderate probability of availability, at the additional cost of more radios. Such results provide some insight for the future network planning using the SDR configuration.

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