Joint Pricing and Resource Allocation for OFDMA-Based Cognitive Radio Systems

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***Abstract*—Cognitive users can share spectrum with primary users under constraints on the interference that results. We present a new pricing strategy for sharing the primary users' available subchannels with cognitive users by optimizing the secondary and primary users' utilities while meeting the primary users' interference constraints. The primary users aim to maximize their revenues by sharing their subchannels with secondary users while ensuring that they achieve a minimum target capacity. On the other hand, the secondary users aim to maximize their capacity under three different constraints: consumed power, a given budget for sharing subchannels, and tolerable interference caused to the primary users. We introduce a sequential procedure based on a distributed algorithm to determine the resource allocation, interference thresholds and prices that satisfy the requirements of both parties in the network. Simulations show that the users face a tradeoff between capacity, power, and price.**

I. INTRODUCTION

According to the measurements of the Federal Communication Commission (FCC), many portions of the radio spectrum are not in use for significant periods of time [1]. Cognitive radio was suggested by Mitola [2] as a solution to increase spectrum efficiency by allowing unlicensed users, called *cognitive or secondary users*, to share these unoccupied frequency bands with their owners, called the *primary users*. The following three scenarios have been suggested for cognitive radio [3]. In the underlay scenario, the secondary users have knowledge of the interference they cause to the primary users and their objective is to transmit without exceeding a fixed interference threshold at the primary users' receivers. In the overlay scenario there is a collaboration between the secondary and primary users via sharing codebooks in order to cancel interference. In the interweave scenario, the secondary users can transmit only in the spectrum holes of the licensed users.

Due to its limited capacity and random availability, the interweave scenario will not suffice to open up significant new spectrum for high-performance applications. In this paper, we consider a situation in which both the interweave and the underlay scenarios are used. Specifically, in our scenario, after spectrum sensing, if the primary channel is unoccupied the secondary users transmit without limit (i.e. the interweave scenario). Otherwise, transmission subject to an interference threshold constraint is considered (i.e. the underlay scenario). Under this model, there are several design questions: How to determine the interference threshold? How to estimate the interference level? What is the benefit to the licensed users in allowing unlicensed users to share their subchannels?

Pricing the shared subchannels was initially proposed by Mitola [4] as a possible benefit for the primary users to share their subchannels with secondary users. Correspondingly, the cognitive users pay the primary users to share their subchannels and can collaborate with them in order to determine an acceptable threshold of interference, time slot, and price on both sides. In the literature, many researchers looked at pricing issues for cognitive radios. For example, in [5], Jayaweera *et al.* developed a game-theoretic framework for dynamic spectrum leasing. In particular, they proposed a generic model for competition between secondary users and showed the existence of an equilibrium. However, this model only considered the use of one primary user in the model. In [6], Niyato and Hossain suggested three different pricing models for different behaviors of the primary users. As such, the problem is treated only from the point of view of the primary users. In [7], Ren and van der Schaar suggested that secondary users can exploit not only the available spectrum but can also use the primary users as relays to forward their packets. This approach needs total availability of the primary users' subchannels for use by secondary users.

In contrast to this prior work, we propose a joint pricing and resource allocation framework for the simultaneous optimization of the primary and secondary users' resources. The secondary users are allowed to share the available spectrum with the primary users under the condition that they do not exceed a fixed interference threshold and they pay a given price. We propose an algorithm to compute the interference threshold level and the price for each subchannel according to the channel conditions and availability in order to maximize the primary users' revenue and the secondary users' capacity.

The remainder of this paper is organized as follows. In Section II, the network model is described. In Section III, the problem is formulated as an optimization problem. In Section IV, the different steps of our proposed approach are presented. In Section V, some selected numerical simulations and results

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are discussed. Finally, in Section VI, conclusions are provided and some future research directions are proposed.

II. SYSTEM MODEL

We consider an orthogonal frequency-division multiple access (OFDMA) based cognitive radio system with L subchannels operating in time division duplex (TDD) mode (i.e. in each subchannel, the same frequency band is used for transmission and reception). A subchannel is a group of subcarriers, possibly correlated, and we assume that the correlation between those subchannels is negligible. We consider a cognitive network composed of N primary users and K secondary users. A user is defined as a transmitter and the corresponding receiver (peer-to-peer mode). Each primary user n owns a set of subchannels $\{L_1, \cdots, L_n\}$ that can be used exclusively for his/her own transmission and/or allowed to be shared by secondary users under a fixed interference constraint that they agree on. The primary users are not supposed to interfere with each other while the secondary users can use subchannels of primary users under the interference constraint mentioned above. In [1], the interference temperature is defined as the radio frequency (RF) power measured at a receiving antenna per unit bandwidth and indicates the tolerable interference level at the primary user. The interference to a different user's receiver is determined by: $P_R = P_T|h_{TR}|^2$, where P_R is the received power, P_T the transmitted power, and h_{TR} the channel gain between the transmitter and receiver. Thus, to protect the primary users, the interference constraint on the power of the secondary users can be written as $P_T \leq \frac{\mathcal{I}}{|h_{TR}|^2}$, where $\mathcal{I}$ is the interference threshold at the primary receiver. The channel condition h_{TR} is estimated based on an average of measurements of the transmitters' and receivers' powers. We suppose that these measurements will be shared between the different users. More specifically, we assume in our model that there is a collaboration between the primary and secondary users in order to make this channel state information (CSI) known to all users. We will show in the next sections how this information can be exploited by the primary and secondary users.

III. PROBLEM FORMULATION

The objective of our work is to efficiently use the frequency spectrum. The primary users tolerate that the secondary users share with them their subchannels without exceeding a specific level of interference. The secondary users pay to the primary network users an amount for each subchannel used. Thus, the primary users' aim is to maximize their profit from selling the unused resources to secondary users after fulfilling their communication requirements. The determined interference threshold should have a minimal effect on the performance of the primary users. The secondary users try to opportunistically utilize the available spectrum holes in order to transmit the maximum amount of data and at the same time pay the minimum price to the primary users for sharing their subchannels. The selection is done per subchannel which means that each subchannel is offered to the secondary users independently.

A. Primary Users' Problem Formulation

From the primary users' point of view, the objective is to maximize the profit from sharing subchannels with secondary users while satisfying their capacity requirement. The objective is to first determine the optimal power and subchannel allocation of the primary users in order to satisfy their own requirements and then to fix the interference threshold levels per subchannel in order to maximize their financial gain from sharing their subchannels with the secondary users.

Mathematically speaking, these can be expressed as the following constrained optimization problem.

For each primary user $n \in \{1, \cdots, N\}$

$$\max \sum_{l=1}^{L} \sum_{k=1}^{K} a_{k,l}^{(c)}\ \mu_l, \quad \text{and} \quad \min \sum_{l=1}^{L} a_{n,l}^{(p)}\ p_{n,l}^{(p)}, \tag{1}$$

$$\text{subject to}$$

$$\sum_{l=1}^{L} a_{n,l}^{(p)} \log_2\big(1 + \frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o + \mathcal{I}_l\ \sum a_{k,l}^{(c)}}\big) \geq \mathcal{C}_n^{min}, \tag{2}$$

$$a_{n,l}^{(p)} \leq b_{n,l}, \quad \forall l \in \{1, \cdots, L\}, \tag{3}$$

$$\textstyle\sum_{n=1}^{N} b_{n,l} = 1, \quad \forall l \in \{1, \cdots, L\}, \tag{4}$$

where

- $a_{n,l}^{(p)}$: Primary user subchannel allocation index matrix.
- $p_{n,l}^{(p)}$: Primary user power allocation matrix.
- $h_{n,l}^{(p)}$: Primary user channel state matrix (PT to PR).
- μ_l : price of sharing the subchannel l.
- $\mathcal{I}_l$: Threshold of interference allowed by primary user to secondary user sharing the subchannel l.
- $\mathcal{C}_n^{min}$: Minimum target capacity for primary user n.

B. Secondary Users' Problem Formulation

From the secondary users' point of view, the objective is to maximize their capacity without exceeding the allowed interference level set by the primary users and using their available budget for buying the subchannels. This objective is met through optimization of the secondary users' power and subchannel allocation to meet these objectives.

For each secondary user $k \in \{1, \cdots, K\}$, this optimization can be expressed as

$$\max \sum_{l=1}^{L} a_{k,l} \log_2\left(1 + \frac{|h_{k,l}^{(c)}|^2 p_{k,l}^{(c)}}{N_o}\right) \tag{5}$$

$$\text{subject to}$$

$$\textstyle\sum_{l=1}^{L} a_{k,l}^{(c)}\ p_{k,l}^{(c)} \leq P_k^{max}, \tag{6}$$

$$\textstyle\sum_{l=1}^{L} a_{k,l}^{(c)}\ \mu_l \leq B_k^{max}, \tag{7}$$

$$\textstyle\sum_{n=1}^{N} a_{n,l}^{(p)}\ a_{k,l}^{(c)}\ p_{k,l}^{(c)}\ |h_{k,l}^{(cp)}|^2 \leq \mathcal{I}_l, \quad \forall l \in \{1, \cdots, L\}, \tag{8}$$

$$\textstyle\sum_{k=1}^{K} a_{k,l}^{(c)} \leq 1, \quad \forall l \in \{1, \cdots, L\}, \tag{9}$$

where

- $a_{k,l}^{(c)}$: Secondary user subchannel allocation index.
- $p_{k,l}^{(c)}$: Secondary user power allocation per subchannel.
- $h_{k,l}^{(c)}$: Secondary user channel state information.
- $h_{k,l}^{(cp)}$: Secondary to primary channel state information.
- P_k^{max} : Power budget for secondary user k.
- B_k^{max} : Secondary user's budget.

Our objective is then to propose subchannel and resource allocation algorithms in order to reach an equilibrium in the subchannel prices and interference levels that satisfy both the primary and secondary users' needs.

IV. PROPOSED JOINT PRICING AND RESOURCE ALLOCATION

Due to the complexity of the problem at hand, we propose a heuristic method to solve this problem based on the following general steps:

1) Primary users preallocate their resources (power and subchannels) in order to to guarantee their own needs;
2) Primary users determine the interference thresholds allowed for secondary users;
3) Secondary users suggest their resource allocation according to the predetermined thresholds;
4) Secondary users bid for the subchannel prices as a function of their budget and the generated capacity;
5) Primary users select the best secondary buyers for each subchannel;
6) Secondary users actualize their power allocation to take into account primary users' decisions.

A. Preallocation of Primary Users' Resources

Since the primary users are the licensed users, they are offered first the opportunity to meet their own requirements. Thus, at the start their resources are allocated to guarantee these requirements. The objective is to minimize their power consumption while meeting their minimum capacity constraint $\mathcal{C}_n^{min}$. The optimization problem can thus be written as

$$\min \sum_{l=1}^{L} b_{n,l}\ a_{n,l}^{(p)}\ p_{n,l}^{(p)}, \tag{10}$$

subject to

$$\sum_{l=1}^{L} a_{n,l}^{(p)}\ \log_2\bigl(1+\frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o+\mathcal{I}_l\ \sum a_{k,l}^{(c)}}\bigr) \geq \mathcal{C}_n^{min}+\epsilon_n. \tag{11}$$

In (11), we added the term ϵ_n as an extra margin for the required capacity $\mathcal{C}_n^{min}$. This margin will allow the tolerance of additional interference from the secondary users while still meeting the primary users' minimum capacity constraint. As proposed in [8], the allocated power is chosen as $p_{n,l}^{(p)} = \frac{N_o(2^{C_{n,l}}-1)}{|h_{n,l}^{(p)}|^2}$, where $C_{n,l}$ is the rate per subchannel computed using a greedy algorithm as per the following formula:

$$C_{n,l} = \left[\rho_n - \log_2\left(\frac{N_o}{|h_{n,l}^{(p)}|^2}\right)\right]^+, \tag{12}$$

where ρ_n is the water level which is determined by assuming that subchannels are sorted in decreasing order of signal-to-noise ratio. This water level is given by

$$\rho_n = \frac{\mathcal{C}_n^{min}+\epsilon_n}{L_n} + \frac{1}{L_n}\sum_{l=1}^{L_n}\log_2\left(\frac{N_o}{|h_{n,l}^{(p)}|^2}\right), \tag{13}$$

where L_n is the number of used subchannels, which is determined by

$$\begin{cases} \frac{\mathcal{C}_n^{min}+\epsilon_n}{L_n} + \frac{1}{L_n}\sum_{l=1}^{L_n}\log_2\left(\frac{N_o}{|h_{n,l}^{(p)}|^2}\right) \leq \log_2\left(\frac{N_o}{|h_{n,L_n}^{(p)}|^2}\right) \\ \frac{\mathcal{C}_n^{min}+\epsilon_n}{L_n+1} + \frac{1}{L_n+1}\sum_{l=1}^{L_n+1}\log_2\left(\frac{N_o}{|h_{n,l}^{(p)}|^2}\right) > \log_2\left(\frac{N_o}{|h_{n,L_n+1}^{(p)}|^2}\right) \end{cases}. \tag{14}$$

B. Interference Threshold Choice

After allocating their own users' resources, the primary networks fix the maximum interference they can tolerate in each subchannel without falling below their required capacity. Specifically, the interference of the secondary users must not violate the constraint presented by the equation:

$$\sum_{l=1}^{L} a_{n,l}^{(p)}\ \log_2\left(1+\frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o+\mathcal{I}_l}\right) \geq \mathcal{C}_n^{min}. \tag{15}$$

This constraint leads to multiple solutions since it contains L unknowns. We present four different scenarios, scenario 1, 2, 3, and 4 below, to overcome this problem and we will compare their performances in our simulations.

Scenario 1: Uniform threshold for all the subchannels

$$\sum_{l=1}^{L} a_{n,l}^{(p)}\ \log_2\left(1+\frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o+\mathcal{I}}\right) \geq \mathcal{C}_n^{min}. \tag{16}$$

Although the maximal threshold cannot be determined explicitly, we prove that the function $f(.)$ defined as $f(\mathcal{I}) = \sum_{l=1}^{L} a_{n,l}^{(p)}\ \log_2\left(1+\frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o+\mathcal{I}}\right) - \mathcal{C}_n^{min}$ is continuous and strictly decreasing and thus it has a unique zero which can be obtained numerically.

Scenario 2: Uniform capacity per subchannel
$\forall l \in \{1, \cdots, L\}$,

$$a_{n,l}^{(p)}\ \log_2\left(1+\frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o+\mathcal{I}_l}\right) = \frac{\mathcal{C}_n^{min}}{\sum_{l=1}^{L} a_{n,l}^{(p)}}. \tag{17}$$

Scenario 3: Capacity of the subchannel with interference proportional to the initial capacity without interference
$\forall l \in \{1, \cdots, L\}$,

$$a_{n,l}^{(p)}\log_2\left(1+\frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o+\mathcal{I}_l}\right) = \mathcal{C}_n^{min}\frac{a_{n,l}^{(p)}\ \log_2\bigl(1+\frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o}\bigr)}{\sum_{l=1}^{L} a_{n,l}^{(p)}\ \log_2\bigl(1+\frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o}\bigr)}. \tag{18}$$

Scenario 4: Profit from channel state information (CSI) between secondary and primary user
Capitalizing on the collaboration between the primary and secondary users to determine the CSI, more interference is

permitted to the useful channels for secondary users in order to increase the primary users' revenues
$\forall l \in \{1, \cdots, L\}$,

$$a_{n,l}^{(p)} \log_2\left(1 + \frac{p_{n,l}^{(p)}|h_{n,l}^{(p)}|^2}{N_o + \mathcal{I}_l}\right) = \mathcal{C}_n^{min} \frac{\min_k h_{k,l}^{(cp)}}{\sum_{l=1}^{L} \min_k h_{k,l}^{(cp)}}. \quad (19)$$

C. Secondary Users' Resource Allocation and Price Bidding

Once the interference thresholds per subchannel are set, the secondary users can compete to offer the best prices to primary users in order to secure their approval of sharing subchannels while maximizing their individual capacity under their power, interference, and budget constraints. Since the secondary users are assumed not to share their channel conditions among themselves, they allocate their powers according to a rate maximization scheme under peak and average power conditions. They can subsequently bid for the price that they are able to pay according to their budget.

1) Resource Allocation: For each user k, the optimization problem can be written as

$$\max \sum_{l=1}^{L} a_{k,l} \log_2\left(1 + \frac{|h_{k,l}^{(c)}|^2 p_{k,l}^{(c)}}{N_o}\right) \quad (20)$$

$$\text{subject to}$$

$$\textstyle\sum_{l=1}^{L} a_{k,l}^{(c)} \; p_{k,l}^{(c)} \leq P_k^{max}, \quad (21)$$

$$\textstyle\sum_{n=1}^{N} a_{n,l}^{(p)} \; a_{k,l}^{(c)} \; p_{k,l}^{(c)} \; |h_{k,l}^{(cp)}|^2 \leq \mathcal{I}_l, \; \forall l \in \{1, \cdots, L\}, \quad (22)$$

This is a convex optimization problem for which the optimal allocation is computed using the standard Lagrangian technique. The resulting power allocation is found as:

$$p_{k,l}^{(c)} = \begin{cases} \left[\dfrac{1}{\rho_k} - \dfrac{N_o}{|h_{k,l}^{(c)}|^2}\right]^+, & l \in \mathcal{U}_c, \\ \min\left\{\left[\dfrac{1}{\rho_k} - \dfrac{N_o}{|h_{k,l}^{(c)}|^2}\right]^+, \dfrac{\mathcal{I}_l}{|h_{k,l}^{(cp)}|^2}\right\}, & l \in \mathcal{U}_p, \end{cases} \quad (23)$$

where $\mathcal{U}_p$ represents the set of subchannels occupied by a primary user so the interference to those users must be taken into account and $\mathcal{U}_c$ are the secondary users' subchannels since the are unoccupied by any primary user. In (23), $\frac{1}{\rho_k}$ is the k-th secondary user's water level which is expressed as

$$\frac{1}{\rho_k} = \frac{1}{|\mathcal{U}_k|}\left(P_k^{(c)} - \sum_{l \in \mathcal{S}_k} \frac{\mathcal{I}_l}{|h_{k,l}^{(cp)}|^2} + \sum_{l \in \mathcal{U}_k} \frac{N_o}{|h_{k,l}^{(c)}|^2}\right), \quad (24)$$

where $\mathcal{U}_k$ and $\mathcal{S}_k$ are, respectively, the set of unallocated subchannels for user k and the set of subchannels whose power should be capped due to interference with primary users.

The water level $\frac{1}{\rho_k}$ is updated iteratively. This can be done using a low complexity iterative algorithm "cap-limited waterfilling" [8], which allows an efficient simultaneous power and subchannel allocation with peak and average power constraints. Note that this resource allocation is independent for each secondary user. As such these allocations can be performed in a parallel and distributed manner.

2) Price Bidding: Using the budget constraint (7), the secondary users determine the price that they can afford for each subchannel. More specifically, the price suggested per subchannel will be proportional to the rate allocated to that subchannel as

$$\mu_{k,l} = B_k^{max} \frac{a_{k,l} \log_2\left(1 + \frac{|h_{k,l}^{(c)}|^2 p_{k,l}^{(c)}}{N_o}\right)}{\sum_{l=1}^{L} a_{k,l} \log_2\left(1 + \frac{|h_{k,l}^{(c)}|^2 p_{k,l}^{(c)}}{N_o}\right)} \begin{cases} \forall l \in \{1, \cdots, L\}, \\ \forall k \in \{1, \cdots, K\}. \end{cases} \quad (25)$$

D. Selection of the Secondary Users per Subchannel

Having the various bids from the secondary users, the primary users select for each subchannel the secondary user who offers the highest price to share their subchannel as

$$k_l = \arg \max_{k \in \{1, \cdots, K\}} \{\mu_{k,l}\}, \quad \forall l \in \{1, \cdots, L\}. \quad (26)$$

E. Allocation Update for Selected Secondary Users

Once the secondary users are selected for each subchannel by the primary users, the secondary users reallocate their powers in their selected subchannels only. In fact, in their initial allocation they divide their powers over all the subchannels but some of these subchannels may end up not being selected because other users may offer higher prices. As such, the secondary users re-run the cap-limited waterfilling but now only over the set of subchannels available for their use.

V. SIMULATIONS AND RESULTS

In this section we demonstrate the performance results of the proposed resource allocation strategy described in the previous section. Extensive simulations were performed, with a system composed of 3 primary users, 4 secondary users, and 64 subchannels equally distributed between the licensed users. The maximal transmit power of each cognitive user was 3 dBm. The required primary users' capacities were generated according to an i.i.d. uniform distribution between 4 and 8 bps/Hz. The noise floor was set to -110 dBm. The primary and secondary users were assumed to be located within a circle of radius 1 Km. The channel state information ($h^{(p)}$, $h^{(c)}$, *and* $h^{(cp)}$) was generated using a Rayleigh model based on the pathloss between the transmitter and the corresponding receiver.

In Fig.1, we see that as we increase the budget of one of the secondary users while fixing the others, its capacity increases but also the amount that this secondary user must pay increases correspondingly. We note that the same behavior will also occur in a similar fashion if the other users increase their budgets, which leads to an interesting competitive game among the secondary users on accessing the available resources that we plan to address in future research. In Fig. 2, we increase gradually the tolerance ϵ_n of one of the primary users while fixing the others. We plot their financial revenues and their consumed powers as functions of this percentage of the tolerance compared to the required capacity. We conclude that this tolerance, ϵ, allows the primary users to control the trade off between power consumption and financial gain from the secondary users. In fact, their power consumption increases exponentially because more power is needed to achieve the new required capacity but at the same time they become more tolerant to secondary users' interference and generate more revenues from sharing subchannels with them. According to the cost of the consumed power and the demand of the secondary users, the primary users can choose the ϵ which maximizes their net revenue. In this figure we also compare the different scenarios, i.e. Scenarios 1, 2, 3, and 4 given above, of threshold selection: while Scenarios 1 and 2 do not exhibit significantly increased revenue from increasing the tolerance, Scenarios 3 and 4 show notable revenue increase with this increased tolerance. Finally, in Fig. 3, we show the effect of the number of secondary users. When this number increases, more competition will be created on the available resources, which leads to a decrease in the individual access to the primary users' resources. However, an increase in the number of secondary users improves the total capacity and revenue since the variability of secondary users (different channel conditions) allows more efficient use of the available spectrum.

VI. CONCLUSION

We proposed a generic framework for pricing the shared subchannels between the primary and secondary users in cognitive radio networks that use a combination of underlay and interweave techniques. The proposed model is characterized by its consideration of both primary and secondary users' performance in terms of capacity, power, and revenue. First, we formulated a generalized optimization problem for primary and secondary user objectives. We then presented a set of distributed algorithms to solve this optimization problem. Extensive simulations were done to show the effect of the different parameters of the problem on the performance of both primary and secondary users. As future work, a game theoretic approach will be suggested in order to choose the best parameters which optimize both primary and secondary users' utilities.

REFERENCES

[1] F. C. Commission, "Spectrum policy task force," Rep. ET Docker no.02-135, Tech. Rep., Nov. 2002.

[2] I. Mitola, J. and J. Maguire, G.Q., "Cognitive radio: Making software radios more personal," *Personal Communications, IEEE*, vol. 6, no. 4, pp. 13 –18, Aug. 1999.

[3] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[4] J. Mitola, "Cognitive radio for flexible mobile multimedia communications," in *the Proceeding of the IEEE International Workshop on Mobile Multimedia Communications (MoMuC '99)*, San Diego, CA, USA, Nov. 1999, pp. 3–10.

[5] S. Jayaweera, K. Hakim, and C. Mosquera, "A game-theoretic framework for dynamic spectrum leasing (DSL) in cognitive radios," in *the Proceedings of the IEEE Global Communications Conference (GLOBECOM'09)*, Honolulu, Hawaii, USA, Nov. 2009, pp. 1–6.

[6] D. Niyato and E. Hossain, "Market-equilibrium, competitive, and cooperative pricing for spectrum sharing in cognitive radio networks: Analysis and comparison," *IEEE Transactions on Wireless Communications*, vol. 7, no. 11, pp. 4273–4283, Nov. 2008.

[7] S. Ren and M. van der Schaar, "Revenue maximization and distributed power allocation in cognitive radio networks," in *the Proceedings of the 2009 ACM Workshop on Cognitive Radio Networks (CoRoNet'09)*, New York, NY, USA, Sep. 2009, pp. 43–48.

[8] N. Papandreou and T. Antonakopoulos, "Bit and power allocation in constrained multicarrier systems: The single-user case," *EURASIP J. Adv. Signal Process*, vol. 2008, Jan. 2008.

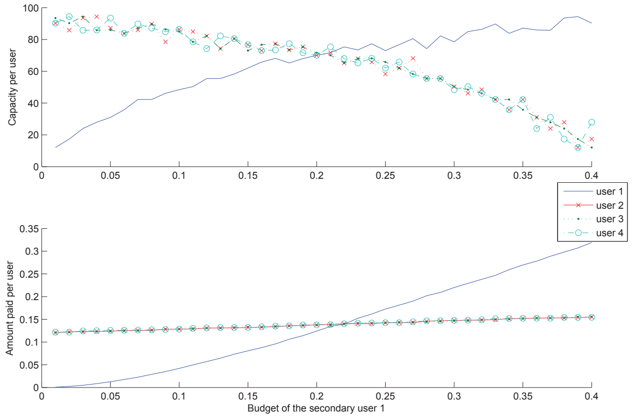

Fig. 1. Capacity and allocated prices variation per secondary user while increasing the budget of the secondary user.

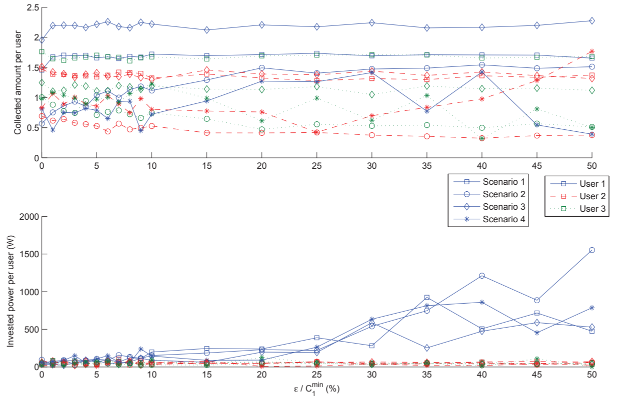

Fig. 2. Financial gain and invested power per primary user as function of the margin ϵ.

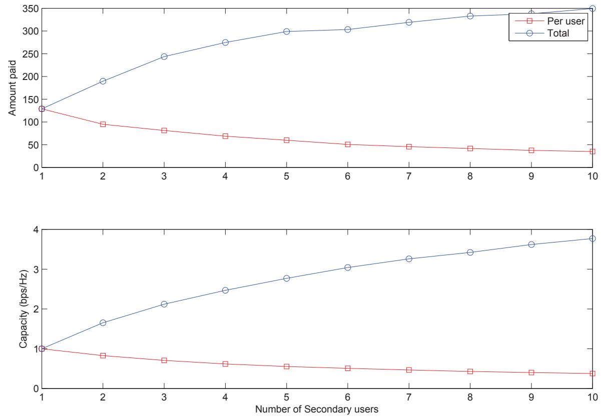

Fig. 3. Effect of the number of secondary users on the total and individual capacity and amount paid.