Optimization Models for Energy Reallocation in a Smart Grid

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***Abstract*—When a malfunction occurs in a Smart Grid electricity provisioning system, it is vitally important to quickly diagnose the problem and take corrective action. The self-healing problem refers to the need to take action in near real time to reallocate power to minimize the disruption. To address this need, we present a collection of integer linear programming (ILP) models designed to identify optimal combinations of supply sources, demand sites for them to serve, and the pathways along which the reallocated power should flow. The models explicitly support the uncertainty associated with alternative sources such as wind power. A simulator configured with multiple intelligent distributed software agents has been developed to support the evaluation of the model solutions.**

Keywords-Smart Grid, resource allocation, self-healing)

I. Introduction

A Smart Grid is a digital age electrical generation and distribution system that is fully networked, instrumented, controlled and automated. A Smart Grid is a quintessential machine to machine system, in which the major components, such as generators, relays, transformers, power lines and electrical meters are networked and digitally addressable with methods such as Internet Protocol (IP) addresses. Many components are also equipped with sensors and processors that are capable of carrying out intelligent actions with little or no human intervention. Available power resources in the Smart Grid include conventional types of generating plants and small-scale renewable Distributed Energy Resources (DERs).

A Smart Grid provides great potential advantages for many stakeholders. At the user level, smart meters at power demand sites open possibilities for dynamic pricing of electricity, making it possible for consumers to receive lower rates by shifting their usage away from periods of high demand to times of low demand. Smart meters also assist utilities by reducing peak loads and allowing them to take action to optimize resource allocation and maximize efficiency. When disruptions occur, instrumentation in the grid immediately communicates exact information that pinpoints the location and type of problem, making maintenance and repair activities more responsive and efficient. At the transmission grid level, Phasor Measurement Units (PMUs) placed at strategic locations provide detailed information about grid health, and can trigger messages that report problems or initiate control actions.

Cascading failures that have occurred in past years highlight the need to understand the complex phenomena that can occur in power networks and to develop emergency controls and restoration procedures. In addition to mechanical failures, overloading a line can create power-supply instabilities such as phase or voltage fluctuations A truly intelligent grid is able to predict impending fault states and failures. [1] [2] [3] [9].

Self-healing capabilities are highly desirable in a smart grid. We define self-healing as the ability to detect the need for corrective actions in the grid, and to autonomously carry out such actions. Once a fault state is detected, the grid itself should perform appropriate procedures, such as dynamically controlling the flow of power to restore grid components from a fault state to normal operation. Examples of common failures that occur in the power grid are power outages, low power quality, overloads that could lead to cascading failures, and service disruptions.

In our work, we model the topology of the Smart Grid as an abstract network of nodes representing supply sources, demand sites, and transshipment junctions, all interconnected by links that represent transmission lines. Devices such as generators, relays, and transformers are associated with specific nodes. Our models are Integer Linear Programs that provide a self-healing capability by identifying optimal alternatives for reallocating and rerouting power when disruptions and failures occur. Failures affect the ability of certain supply sources to meet demands for energy at certain demand sites. Our primary modeling goal is to balance the flow of power across the system to ensure that no consumer site experiences an outage, while also maximizing the overall efficiency, cost effectiveness, and reliability of the system. Our models account for multiple factors, such as availability, reliability, uncertainty, cost-effectiveness and consumer preference. The basic modeling template is the Capacitated Transshipment Problem (CTP). Additional model structure incorporates uncertainty at supply sources and ensures that capacities (load limits) on transmission lines and through devices are not exceeded. Uncertainty of available supply at certain sources is modeled within the integer linear programming framework using chance-constrained programming methods. The integer linear programming models provides the basis for intelligent decision making in the grid as it pertains to resource allocation.

An agent-oriented simulation of Smart Grid operation is available to test and evaluate alternative resource allocation solutions.

This paper is organized in four sections following the Introduction. Section II Provides a problem statement and necessary background. Section III provides a brief review of Smart Grid modeling and Integer Linear Programming (ILP). In Section IV we present a collection of ILP models that capture various aspects of the self-healing problem, including an uncertainty model. Section V discusses the evaluation of the integer linear programming models in a smart grid simulation environment. In section VI we present conclusions and describe future work.

II. PROBLEM STATEMENT

In building a Smart Grid self-healing model, there are multiple issues that are important to include. Some pertain to the physical infrastructure, such as the generators, busses, relays, and transmission lines. Other considerations pertain to the cyber infrastructure, such as the communication networks, storage, protocols, security, and procedures for management of the grid. Here we focus on the following issues in the physical infrastructure that involve resource allocation.

- Distributed Device Control Functions. Most devices associated with nodes in the system must be controllable through remote action. One example is the traditional remote relay control circuit that is capable of tripping a circuit breaker under conditions of electrical current higher than a threshold. A second example is adaptive control of inverters to ensure stable voltages. Fully centralized control is impossible, and local device control with distributed intelligence is highly desirable.

- Selective Load Control. The ability to selectively switch off customers under certain conditions can help avoid a wide-ranging blackout. This also allows consumers to contract to manage their energy consumption, emphasizing low-cost time periods.

- Micro-grid Islanding. Distributed Energy Resources (DERs) are small-scale power generators such as micro turbines, diesel generators, solar arrays, fuel cells, and wind farms that are located near a customer cluster. When configured into a micro grid, these systems typically automatically disconnect themselves from a single point of connectivity with the primary grid when a disruption occurs. When the primary grid is returned to normal conditions, a micro-grid must reconnect and resynchronize its operation.

- Distributed Pathway Control. The use of alternative redundant pathways for electricity can be utilized to maintain service under disruptive conditions.

The mathematical models we develop are focused on the distributed pathway control issue, with an objective of finding an optimal set of alternative pathways for electricity to flow from supply sources to demand sites, while also satisfying constraints on transmission line capacity.

III. SMART GRID MODELING

Several models have been developed to characterize the functioning of the grid under various conditions. A probabilistic model of load dependent cascading failure is presented in [5] and [6]. The important area of managing consumer consumption of electricity in response to supply conditions and pricing has drawn attention. The role of factors such as load scheduling and market prices in driving consumer behavior and achieving energy efficiency is described in [10] and [11]. In [11], user preferences are modeled using the concept of discomfort level within an optimization problem formulation that balances the load and minimize user inconvenience caused by demand scheduling. In [12], an energy consumption scheduling problem is established to minimize the overall energy cost. Javed et al. [15] formulated a linear program for distribution management. Kadar [14] developed an optimization model for the design of Smart Grid network infrastructure. Our work is the first development of optimization models specifically for real-time self-healing, and that directly incorporates uncertainty.

At the center of any power system design is the control and communication architecture, comprising the hardware and protocols for exchanging critical status and control signals. In conventional electric power systems, this is the responsibility of the Supervisory Control and Data Acquisition (SCADA) system [7] [8].

IV. INTEGER LINEAR PROGRAMMING MODELS

Early Linear Programming (LP) models came into prominence and practice during World War II as a means to improve efficiency and utilization of scarce resources. LP models have a linear objective function to minimize or maximize and linear constraints in the form of equations or inequalities. The Simplex method developed by Dantzig [16] has been a mainstay solution methodology, and the more recent interior point method is also prominent. Integer Linear Programming (ILP) models often arise from node-arc network formulations. Network models of this type date to the pioneering work of Ford and Fulkerson [17]. The work in [18] on the Capacitated Transshipment Problem (CTP) first gave full descriptions of highly efficient solution algorithms for the type of ILP that applies to the self-healing problem.

In a self-healing Smart Grid, we assume that disruptions in energy availability occur due to such things as malfunctioning or failed devices and/or inoperative transmission lines. These disruptions affect the ability of specific supply sources to meet demands for energy at specific demand sites. In response to the associated need to allocate electrical power in alternative ways to accomplish self-healing, we devise several optimization models of increasing complexity to assign supply sources to demand sites. More specifically, assume that there are J distinct demands for energy for which alternative sources of supply must be allocated in the short term to respond to disruptions. For each of these J demands there is a finite set of available supply sources that can be allocated to meet the demand. We index the supply sources by i = 1, 2, 3, …,I. The

Sponsor: U. S. Department of Energy Grant DE-FG52-08NA28921

following figure shows a bipartite graph in which the supply sources are nodes in the left set and demand sites are nodes in the right set.

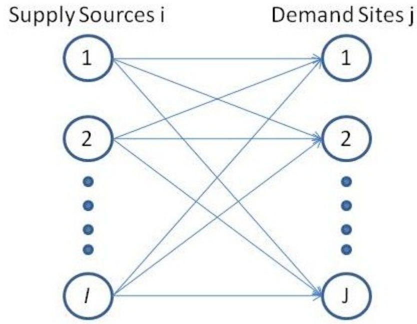

Figure 1: A bipartite graph with supply and demand nodes

The arcs in the graph model intact transmission paths with possibly multiple links that utilize sequences of transmission lines, busses, relays, transformers, capacitors, and other devices. The graph is typically not complete, with missing arcs modeling the unavailability of a viable transmission path. We use c_{ij} to denote the cost of assigning supply source i to demand site j. The objective function parameters are evaluations of a utility function that includes multiple factors taken together, such as prices established under existing contracts, regulatory principles, prices negotiated in near real time, issues related to the viability of transmission paths, and expected reliability. A given supply source i has a specified level s_i of energy available, demand sites have a specified level d_j of energy needed, sources can supply multiple demand sites, and demand sites can be served from multiple sources. We note that available supplies and demands can be split freely in their allocations, and the variables x_{ij} can be viewed as flows of power from supply sources to demand sites. We also must ensure that the transmission paths that connect supply sources to demand sites have sufficient capacity to bear the load level placed on them. In a self-healing situation, we let u_{ij} denote the increase in load level (capacity) that can be allocated to an available pathway connecting nodes i and j. This leads to the following problem:

$$Max\ z = \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}\, x_{ij}$$

Subject to:

$$\sum_{i=1}^{I} x_{ij} \geq d_j \qquad for\ all\ j$$

$$\sum_{j=1}^{J} x_{ij} \leq s_i \qquad for\ all\ i$$

$$0\ \leq x_{ij}\ \leq\ u_{ij} \qquad for\ all\ i\ and\ j$$

One limitation of this basic model is the implicit assumption that the transmission paths modeled by the arcs have no links in common, which may not be the case in practice. This leads to an expanded model formulation that breaks the bipartite graph into a more general network and includes capacities on individual links.

Notation:

(i) The directed graph (network) has node set N and link set A = N x N. We denote typical elements: i ε N, (i,j) ε A

(ii) c_{ij} = utility per power flow unit on (i,j)

(iii) u_{ij} = capacity (upper bound) of (i,j)

(iv) b_i = supply of power at node i (interpret negative b_i as a demand of $-b_i$ units)

(v) Variables x_{ij} = power flow on link (ij).

The problem is to find the set of flows that minimize total cost subject to constraints which require i) "flow balance" at each node and ii) capacity restriction on each link. The formulation follows.

$$Max\ z = \sum_{(i,j)\in A} c_{ij}x_{ij}$$

Subject to:

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = b_i \qquad for\ all\ i$$

$$0 \leq x_{ij} \leq u_{ij} \qquad for\ all\ i\ and\ j$$

The first constraint set consists of flow balance conditions. The first term in such a constraint is summed over all links with tail at node i, referred to as the “forward star” of node i. Similarly, the second term is summed over all links with head at node i, the "reverse star" of node i. This model requires that total supply and total demand be equal, a condition easily enforced through the use of dummy nodes as needed. The second set of constraints enforces capacity limitations. The model is known as the Capacitated Transshipment Problem (CTP) in the literature. Figure 2 illustrates the topology of this type of network.

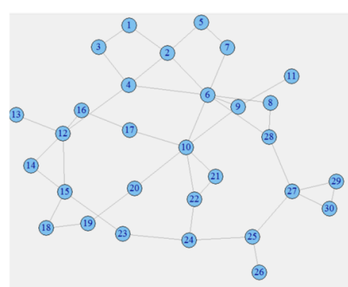

Figure 2. Smart Grid network topology

More generally, it may be important to explicitly distinguish among supply sources by type. For example, if a site supplies power from wind, there may be specific important information about that source, such as uncertainty. In the following model, supply sources and demand sites are indexed and differentiated by type p, where the index takes on values p = 1,2,…,P. Accordingly, we now have the notation and model given below.

Parameters:
c_{ijp} = utility per unit of flow of type p on link (i,j)
u_{ij} = capacity (upper bound) for flow on link (i,j)
b_{ip}= supply of power of type p at node i (interpret negative b_i as a demand of $-b_i$)
Variables:
x_{ijp} = flow of power of type p on link (ij).

$$Max\ z = \sum_{(i,j)\in A} \sum_{p\in P} c_{ijp}\, x_{ijp}$$

Subject to:

$$\sum_{j:(i,j)\in A} x_{ijp} - \sum_{j:(j,i)\in A} x_{jip} = b_{ip} \qquad for\ all\ i\ and\ p$$

$$\sum_{p=1}^{P} x_{ijp} \le u_{ij} \qquad for\ all\ i\ and\ j$$

$$x_{ijp} \ge 0 \qquad for\ all\ (i,j) and\ p$$

In the literature this is known as the multi-commodity CTP. The first constraint set enforces that flow balance must occur for each type of power through every node i. The value of b_{ip} is positive at strictly supply source nodes, negative at strictly demand site nodes, and zero at pure transshipment nodes. The model allows for supply sources or demand sites to also serve as transshipment points, but this would be unusual in practice. The second constraint set allows for each link in the distribution system to be restricted by joint capacity over all flows that pass through it. The model is NP-complete.

We now consider the possibility that supplies and demands at certain nodes are uncertain, such as is often the case for supply sources like wind power or solar. The typical power curve in Figure 3 illustrates the uncertainty of power output obtainable from a wind machine.

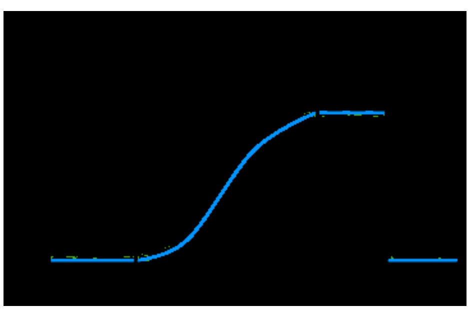

Figure 3: Wind Machine Power Curve

For a given source node i and power type p we modify a constraint in the first set above to make it probabilistic as follows:

$$\Pr\left[\sum_{j:(i.j)\in A} x_{ijp} = b_{ip}\right] \ge 1 - \propto_{ip} \quad for\ a\ specific\ i\ and\ p$$

For ease of exposition, here we assume that node i is the sole source of commodity type p and that it does not serve as a transshipment point for power originating at other sites. In this constraint, $1 - \alpha_{ip}$ is a pre-assigned smallest allowable probability with which the power available from the source i is sufficient to supply b_{ip} units to a demand site. We view α_{ip} as the acceptable risk of not receiving b_{ip} MW of electrical power from the specific DER source. For specific values of i and p, we assume that b_{ip} is a random variable that follows a statistical distribution. We note that variation in the value of b_{ip} in turn results in variation of the flows through the links in the network which then in turn also affects the capacity constraints for the links. In the case where b_{ip} follows the Normal distribution with mean $E\{b_{ip}\}$ and variance $Var\{b_{ip}\}$ we standardize the random variable by subtracting the mean and dividing by the square root of the variance, resulting in the following equivalent probabilistic condition:

$$Pr\left[\left[\frac{\sum_{j:(i,j)\in A} x_{ijp} - E\{b_{ip}\}}{\sqrt{Var\{b_{ip}\}}}\right] = \left[\frac{b_{ip} - E\{b_{ip}\}}{\sqrt{Var\{b_{ip}\}}}\right]\right] \ge 1 - \alpha_{ip}$$

The true meaning of the equation is that the enforcing of the condition that the power distributed from supply source i to its outgoing links should be at a level for which there is confidence that at least that much power will actually be delivered with a prescribed probability. Any overage would likely be dissipated. This consideration makes it legitimate to replace the equation with an inequality in the analysis:

$$Pr\left[\left[\frac{\sum_{j:(i,j)\in A} x_{ijp} - E\{b_{ip}\}}{\sqrt{Var\{b_{ip}\}}}\right] \ge \left[\frac{b_{ip} - E\{b_{ip}\}}{\sqrt{Var\{b_{ip}\}}}\right]\right] \ge 1 - \alpha_{ip}$$

We let Φ represent the cumulative distribution function for the standard normal distribution and let $K_{\propto_{ip}}$ be the standard normal value such that $\Phi\left(K_{\propto_{ip}}\right) = 1 - \propto_{ip}$ for significance level $\propto_{ip}$. The probabilistic condition is then realized if

$$\left[\frac{\sum_{j:(i,j)\in A} x_{ijp} - E\{b_{ip}\}}{\sqrt{Var\{b_{ip}\}}}\right] \ge K_{\propto_{ip}}$$

This can be rewritten as a constraint

$$\sum_{j:(i,j)\in A} x_{ijp} \le E\{b_{ip}\} + K_{\propto_{ip}}\sqrt{Var\{b_{ip}\}}$$

This constraint gives the condition that the power delivered will be within the upper bound value given by the right-hand side with probability $1 - \alpha_{ip}$. By the symmetry of the Normal distribution, we get the following constraint:

$$\sum_{j:(i,j)\in A} x_{ijp} \ge E\{b_{ip}\} - K_{\propto_{ip}}\sqrt{Var\{b_{ip}\}}$$

This condition sets the requirement for the minimum level of power that will be delivered with the prescribed probability. This is a linear constraint that is incorporated into the optimization problem as a so-called "chance constraint," effectively modeling probabilistic conditions within a linear program. As an example, suppose that the supply source at node 3 is a wind source that provides power with a mean value of 7 MW and a variance of 4 MW^2 and has outgoing distribution links to transshipment nodes 4, 7, and 8. Node index 3 also identifies the type of power generated at node 3. If we allow a 5% risk of not meeting the supply objective, we have the following condition:

$$x_{343} + x_{373} + x_{383} \leq 7 + 1.645 * 2$$

or

$$x_{343} + x_{373} + x_{383} \leq 11.935$$

The value 1.645 comes from a table of standard normal variates. The condition means that there is a 95% chance that the realizable power from the wind source is no more than 11.935 MW. Using the symmetry,

$$x_{343} + x_{373} + x_{383} \geq 7 - 1.645 * 2$$

or

$$x_{343} + x_{373} + x_{383} \geq 3.71$$

This means that at least 3.71 MW of power can be realized with 95% probability. If we increase the prescribed probability to a more stringent 99%, the standard Normal variate value is 2.33 and the constraint becomes

$$x_{343} + x_{373} + x_{383} \geq 2.34$$

The model can also be readily extended into multiple time periods with time-indexed supply-demand allocation with fixed costs. This is important for consistency with time period planning granularity models used by most utilities.

V. SMART GRID SIMULATION

Our smart grid simulator runs as a Multi-Agent System (MAS) using the Java Agent Development Framework (JADE). Software agents act autonomously and communicate with each other across open and distributed environments, which is ideal design for simulating a Smart Grid. The agents sense, act, communicate and collaborate with each other, are empowered with degrees of autonomy, are decentralized, and have local views and knowledge. The simulation has a low-level physical device layer with components that can exhibit fault conditions and fail. There are intelligent middle layer agents for consumers, DERs, devices, managers, and monitors. An upper layer consists of management agents that receive system state information, carry out analyses, and invoke decision support models. The optimization models described in this paper reside at this third level. However, the simulator also supports suites of decision models, including fuzzy logic, statistical hypothesis testing, and Bayesian belief networks. These agents also stream reporting information, allowing for convenient comparison of model performance.

When a third layer optimization model generates a workable solution in a self-healing situation, it is converted into the associated corrective actions that are carried out at lower layers to invoke the appropriate response. Each corrective action consists is modeled by an agent/task pair. The task breaks out into detailed roles and actions at the level of the devices and transmission lines. A graphical user interface allows human intervention if appropriate or autonomous execution by simply setting initial values for parameters conditions, and state information.

VI. CONCLUSIONS

The optimization models developed include objective functions that optimize a utility function, and constraints that ensure feasibility of the resource allocations. Stochastic information can be directly included in the constraints, to model situations with known uncertainty. The agent-based simulation provides a realistic and readily validated means of evaluating the performance of the integer linear programming solutions as they would function in an operational Smart Grid.

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