

Distributed Compressed Wideband Sensing in Cognitive Radio Sensor Networks

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Abstract—A novel distributed compressed wideband sensing scheme for Cognitive Radio Sensor Networks (CRSN) is proposed in this paper. Taking advantage of the distributive nature of CRSN, the proposed scheme deploys only one single narrowband sampler with ultra-low sampling rate at each nodes to accomplish the wideband spectrum sensing. First, the practical structure of the compressed sampler at each node is described in detail. Second, we show how the Fusion Center (FC) exploits the sampled signals with their spectrum randomly-aliased to detect the global wideband spectrum activity. Finally, the proposed scheme is validated through extensive simulations, which shows that it is particularly suitable for CRSN.

I. INTRODUCTION

Many advanced sensor network applications require that the sensed information are delivered in the form of broadband multimedia by resource-constrained sensor nodes, or need the sensor nodes to be survivable in highly dynamic spectrum environments. This is hard to achieve in traditional wireless sensor networks (WSN), since they usually compete for limited available bandwidth on fixed unlicensed bands. A promising sensor networking solution for these applications is the newly introduced paradigm of Cognitive Radio Sensor Networks (CRSN) [?], which incorporates cognitive radio (CR) capability into the traditional WSN. Without causing harmful interference to the primary user (PU) systems, CRSN can provide dynamic spectrum access and the bandwidth as high as possible to meet the application-specific QoS requirement.

According to the NSF Spectrum Occupancy Measurements [?], the spectrum occupation pattern is sparse. Exploiting this sparse prior, several compressed wideband sensing schemes are developed based on the literatures of compressed sensing (CS) [?]. Tian developed a distributed compressed spectrum sensing approach for wideband CR networks [?]. Her scheme can achieve high-performance at low sampling rate below the Nyquist rate. Several other works [?]-[?] have also studied the application of CS on wideband spectrum sensing. However, to the best of our knowledge, their emphasis is not on the specific sampling methods, i.e. little information is provided about the structure of the sampler and their feasibility in practical

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implementation. Moreover, their sampling rate for individual CR should exceed the sum of the active bandwidth, which is still too high for the energy-constrained CRSN nodes.

In this paper, we propose a novel distributed compressed wideband sensing scheme for CRSN. The main contributions of this paper are two-fold. First, we propose practical wideband sampling structure in fine detail. Second, the required sampling rate in our scheme is equivalent to the bandwidth of a single subband, much lower than existing schemes. Our sampling structure is similar to the Sub-Nyquist sampler proposed recently by Mishali and Eldar - the modulated wideband converter (MWC) [?]. The major difference is that we only deploy one sampling channel in a single CRSN node, and each node randomly mixes the signals of all subbands by spectrum aliasing. This simple structure is extremely convenient for implementation in the resource-constrained CRSN, due to its low hardware cost and power consumption.

The rest of this paper is organized as follows. In Section II, we establish basic models for the PU signal and the CRSN, and formulate our target problem. In Section III, we introduced the distributed compressed wideband sensing scheme in detail. Then, extensive simulation results are presented in Section IV to further validate our proposed algorithm. Finally, the whole paper is concluded in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the radio environment to be an ultra-wide frequency band where different PU systems and large numbers of CRSN nodes coexist.

A. Primary User Signal

The entire W wide spectrum is centered at zero and can be divided into L non-overlapping subbands of equal bandwidth B , thus $W = L \cdot B$. We assume there are J PUs occupying some of the subbands. According to [?], the PU signal is sparse within the range of the whole spectrum. Therefore, we can safely assume that J is much smaller than L . Note that we will take this assumption as our basic prerequisite throughout this paper.

We assume real-valued continuous-time wideband signal of the primary users to be $x(t)$, which is band-limited within $(-\frac{W}{2}, \frac{W}{2})$ and its Fourier transform can be defined by

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft} dt \quad (1)$$

As mentioned above, the frequency domain is zero centered and divided into L subbands of equal bandwidth B , $X(f)$ can be re-written as

$$X(f) = \sum_{l=-L_0}^{+L_0} X_l(f) \quad (2)$$

where $L = 2L_0 + 1$ and $X_l(f) = \begin{cases} X(f), & f \in ((l - \frac{1}{2})B, (l + \frac{1}{2})B) \\ 0, & \text{otherwise} \end{cases}$ is the signal in the l th subband of the wideband spectrum. Thus, we can represent the primary signal using a vector $\mathbf{X}(f) = [X_{L_0}(f), \dots, X_0(f), \dots, X_{-L_0}(f)]^T$. Since $x(t)$ is real-valued continuous-time signal, the real part of Fourier transform $X(f)$ is an even function, and the imaginary part is an odd function. Then, obviously $|X_l(f)| = |X_{-l}(f)|$.

B. Secondary User System: CRSN nodes and Fusion Center

The Secondary User (SU) system is comprised of CRSN nodes and Fusion Centers. The spatially distributed CRSN nodes can be wireless terminals such as webcams, cellphones and laptops. Their application-specific QoS requires that these CRSN nodes deliver event features reliably and timely in the form of multimedia, resulting in high bandwidth demands. Therefore, our objective is to design a resource-efficient wideband spectrum sensing scheme for the CRSN.

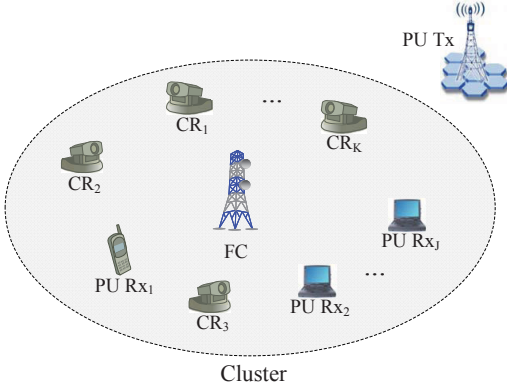


Fig. 1. An overview of the considered system model.

As shown in Fig.??, CRSN nodes located in a restricted locality form a cluster. All nodes in one cluster sense the wideband cooperatively. There is a Fusion Center (FC) in each cluster, which is a special kind of node equipped with more or renewable power sources. As a result, FC have additional computation capability and longer transmission ranges. Since all the CRSN nodes in a cluster are close to each other, we assume that a common control channel can be found to exchange spectrum sensing information.

Suppose that there are K CRSN nodes engaged in the wideband spectrum sensing task. Because these K nodes are close to each other, the PU activities in their ambient radio environment are assumed to be identical. However, due to different channel fading, the received primary signal can be

different at each CRSN nodes. During the sensing interval, the primary signal received by the k th CRSN nodes can be expressed by

$$X^k(f) = X(f) H^k(f) + W^k(f) \quad (3)$$

Using $\mathbf{H}^k(f) = [H_{L_0}^k(f), \dots, H_0^k(f), \dots, H_{-L_0}^k(f)]^T$ and $\mathbf{W}^k(f) = [W_{L_0}^k(f), \dots, W_0^k(f), \dots, W_{-L_0}^k(f)]^T$ to represent channel fading and noise of all subbands, we can rewrite (3) in the matrix form

$$\mathbf{X}^k(f) = \text{diag}(\mathbf{H}^k(f)) \mathbf{X}(f) + \mathbf{W}^k(f) \quad (4)$$

III. DISTRIBUTED COMPRESSED WIDEBAND SPECTRUM SENSING SCHEME

In this section, we propose a cooperative spectrum sensing scheme that can effectively distribute the sensing task to many CRSN nodes and relieve the sensing and processing burden on a single node. We first describe the compressed aliasing sampling scheme originated from the Modulated Wideband Converter (MWC), then explain the fusion strategy and corresponding recovering algorithm.

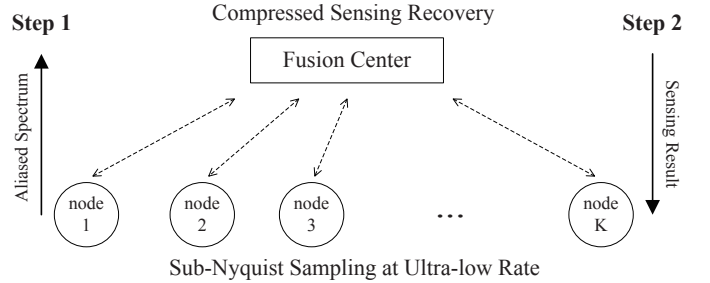


Fig. 2. Distributed Compressed Wideband Sensing Scheme.

As shown in Fig.2, our proposed sensing scheme comprises two steps.

Step 1: K CRSN nodes sample the wideband independently at ultra-low rate equivalent to the bandwidth of a single subband, then report the aliased spectrum information to the FC.

Step 2: After receiving all the aliased spectrum information from the K CRSN nodes, the the FC performs compressed sensing recovery algorithm, and broadcast the sensing result to all the CRSN nodes within the cluster.

A. Distributed Sampling based on Spectrum Aliasing

The sampling method is similar to the Modulated Wideband Converter(MWC) [?][?]. The difference is that we only deploy one channel of the MWC system in a single CRSN node. The sampling structure in the k th CRSN node is shown in Fig.3.

According to Fig.3, after the primary signal $x^k(t)$ arrives at the k th CRSN node, it is multiplied by a high frequency mixing function $p^k(t)$. $p^k(t)$ is a T_s -length periodic spread-spectrum mixing function which aims to alias the spectrum

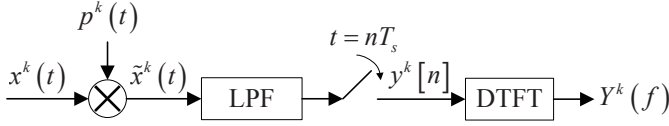


Fig. 3. Compressed sampling structure based on aliasing for a CRSN node.

and thus obtain a mixture of signals from all the subbands.

$$p^k(t) = \alpha_{km}, m \frac{T_s}{M} \leq t \leq (m+1) \frac{T_s}{M}, 0 \leq m \leq M-1 \quad (5)$$

where $\alpha_{km} \in \{+1, -1\}$, and $p^k(t) = p^k(t + nT_s)$ for every $n \in \mathbb{Z}$. In practice, $p^k(t)$ is characterized by pseudorandom sequences generated by certain seeds known at the FC. Therefore, the aliasing pattern of the spectrum is known at the FC.

Then, the mixed analog spectrum signal goes through a Low-Pass Filter (LPF) with cutoff frequency $1/2T_s$ and the filtered signal is sampled at rate $f_s = 1/T_s$. For convenient spectrum access, we choose f_s to be equal to B , so low that existing A/D converters are competent for the task.

We perform discrete-time Fourier transform (DTFT) on the sampled data, in order to obtain the frequency domain form of the mixed spectrum. According to [?],

$$Y^k(e^{j2\pi f T_s}) = \sum_{l=-L_0}^{+L_0} c_l^k X^k(f - lB) \quad (6)$$

where $f \in \mathcal{F}_s = [-B/2, +B/2]$, $L_0 = \lceil \frac{W-B}{2B} \rceil$, and $c_l^k = B \int_0^{1/B} p^k(t) e^{-j \frac{2\pi}{T_s} l t} dt$ is the Fourier coefficient of $p^k(t)$. After a closer examination, we can easily find that (6) is a weighted sum of the spectrum of all subbands, and can be rewritten in the matrix form:

$$Y^k(f) = \mathbf{c}^k (\text{diag}(\mathbf{H}^k(f)) \mathbf{X}(f) + \mathbf{W}^k(f)) \quad (7)$$

where $\mathbf{c}^k = [c_{L_0}^k, \dots, c_{-L_0}^k]$.

At last, we calculate the average level of the band-limited spectrum mixture, and transmit this aliased spectrum information to the FC through the control channel.

$$Y^k \equiv \bar{Y}^k(f) = \int_{-B/2}^{+B/2} |Y^k(f)| df \quad (8)$$

B. Fusion Strategy and Recovering Algorithm

K CRSN nodes engage in the spectrum sensing task and report their spectrum mixture value to the fusion center, all the reported value can compose a K -length vector $\mathbf{Y} = [Y^1, \dots, Y^K]^T$

Here, we further use the assumption that all the K CRSN nodes and the FC are close to each other and their channel fadings are approximately the same, i.e. $\mathbf{H}^1(f) = \dots = \mathbf{H}^k(f) = \mathbf{H}(f)$. It can be estimated at the FC and broadcasted to all the CRSN nodes.

The K -length vector collected from K distributed CRSN nodes to the FC can be expressed as

$$\begin{aligned} \mathbf{Y} &= \int_{-B/2}^{+B/2} |\mathbf{C}(\text{diag}(\mathbf{H}(f)) \mathbf{X}(f) + \mathbf{W}(f))| df \\ &= \int_{-B/2}^{+B/2} |\mathbf{A}(f) \mathbf{X}(f) + \mathbf{W}'(f)| df \end{aligned} \quad (9)$$

where \mathbf{C} is the Fourier coefficient matrix of mixing functions $\mathbf{p}(t)$, $\mathbf{A}(f)$ is a $K \times L$ measurement matrix, $\mathbf{A}_{kl}(f) = c_l^k H_l(f)$ and $\mathbf{W}'(f)$ is also a noise vector.

Assuming that the channel fading are also almost flat within a single subband, i.e. $H_l \equiv \bar{H}_l(f) = \int_{-B/2}^{+B/2} |H_l(f)| df$, equation (9) can be rewritten as

$$\mathbf{Y} = \mathbf{A} \mathbf{X} + \mathbf{W}' \quad (10)$$

where $\mathbf{A}_{kl} = c_l^k H_l$, and we denote $\mathbf{X} = \int_{-B/2}^{+B/2} |\mathbf{X}(f)| df$,

$\mathbf{W}' = \int_{-B/2}^{+B/2} |\mathbf{W}'(f)| df$ to be the average level of the primary signal and noise, respectively.

Based on the analysis in [?], \mathbf{A} can be expressed as (11) (on the top of next page), where $\theta = e^{-j \frac{2\pi}{T_s} l t}$ and

$$d_l = \frac{1}{T_s} \int_0^{T_s} e^{-j \frac{2\pi}{T_s} l t} dt = \begin{cases} \frac{1}{L} l = 0 \\ \frac{1 - \theta^l}{j 2\pi l} l \neq 0 \end{cases} \quad (12)$$

Considering (10), where \mathbf{X} is an unknown sparse vector of dimension L , \mathbf{Y} is the measured vector of dimension K , and \mathbf{A} is the known random measurement matrix of size $K \times L$. Our final goal is to recover the original primary signal \mathbf{X} from the measured compressed vector \mathbf{Y} and determine the idle subbands for opportunistic spectrum access. This is exactly the compressed sensing (CS) problem [?] since $K \ll L$.

It is proved that a random sign matrix, whose entries are drawn independently from ± 1 with equal probability, has the RIP of order S if $K \geq C \cdot S \log(L/S)$ [?][?], where C is a positive constant. Also, any fixed unitary row transformation of a random sign matrix has the same RIP [?]. From (12), we can see $\mathbf{S}_{K \times L}$ is a random sign matrix, this implies that $\mathbf{A}_{K \times L}$ also has RIP and (10) can be solved using existing CS recovery methods.

In this paper, we employ the basis pursuit (BP), a convex programming method that can solve CS problems in polynomial-time. We choose BP for the following two reasons. First, the sparsity level of the input signal is known in other OMP [?] related CS recovery algorithm. However, in our scenario, the sparsity level means the number of the active subbands and can not be estimated beforehand. Second, the BP algorithm has robust performance in the noisy scenario.

We first estimate global spectrum vector $\hat{\mathbf{X}}$ through l_1 minimization based noisy BP algorithm:

$$\min \|\hat{\mathbf{X}}\|_{l_1} \quad \text{subject to} \quad \|\mathbf{A} \hat{\mathbf{X}} - \mathbf{Y}\|_{l_2} \leq \sigma_W^2 \quad (13)$$

$$\begin{aligned}
\mathbf{A}_{K \times L} &= \begin{pmatrix} \alpha_{1,0} & \cdots & \alpha_{1,L-1} \\ \vdots & \ddots & \vdots \\ \alpha_{K,0} & \cdots & \alpha_{K,L-1} \end{pmatrix} \begin{pmatrix} \theta^{L_0 \cdot 0} & \cdots & \theta^{-L_0 \cdot 0} \\ \vdots & \ddots & \vdots \\ \theta^{L_0 \cdot (L-1)} & \cdots & \theta^{-L_0 \cdot (L-1)} \end{pmatrix} \begin{pmatrix} d_{L_0} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d_{-L_0} \end{pmatrix} \begin{pmatrix} H_{L_0} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_{-L_0} \end{pmatrix} \\
&= \mathbf{S}_{K \times L} \times \mathbf{F}_{L \times L} \times \mathbf{D}_{L \times L} \times \mathbf{H}_{L \times L}
\end{aligned} \tag{11}$$

where σ_W^2 is the variance of the Gaussian noise W' .

And then define a decision vector $\hat{\mathbf{d}}$ of the PU activity state by comparing the l th subband estimation \hat{X}_l with a decision threshold λ :

$$\hat{\mathbf{d}} = [\hat{d}_{L_0}, \dots, \hat{d}_l, \dots, \hat{d}_{-L_0}] \tag{14}$$

$$\text{where } \hat{d}_l = \begin{cases} 0, & \hat{X}_l < \lambda \\ 1, & \hat{X}_l > \lambda \end{cases}$$

The decision vector serves as our compressed wideband spectrum sensing result. After this decision vector is obtained at FC, this final result is broadcasted to all the CRSN nodes within the cluster. The CRSN nodes then access the spectrum according to this sensing result.

IV. PERFORMANCE EVALUATION

To evaluate the accuracy of our proposed compressed sensing recovery algorithm, we define the normalized root mean-square estimation error (MSE):

$$\text{MSE} = \mathbb{E} \left(\frac{\|\hat{\mathbf{X}} - \mathbf{X}\|_{l_2}}{\|\mathbf{X}\|_{l_2}} \right) \tag{15}$$

Just like any cognitive radio network, we choose the probability of detection P_d and the probability of false alarms P_f as our wideband sensing performance metrics. In the multi-band scenario, they can be expressed as

$$\begin{aligned}
P_d &= \mathbb{E} \left(\frac{\text{Num}_{l=L_0, \dots, -L_0} (d_l=1 \text{ and } \hat{d}_l=1)}{\text{Num}_{l=L_0, \dots, -L_0} (d_l=1)} \right) \\
P_f &= \mathbb{E} \left(\frac{\text{Num}_{l=L_0, \dots, -L_0} (d_l=0 \text{ and } \hat{d}_l=1)}{\text{Num}_{l=L_0, \dots, -L_0} (d_l=0)} \right)
\end{aligned} \tag{16}$$

where $d_l = \begin{cases} 0, & \text{the } l\text{th subband is idle} \\ 1, & \text{the } l\text{th subband is busy} \end{cases}$ represents the PU activity pattern on the spectrum.

In the following experiments, we set the system parameters to be an ultra-wideband regime. For all scenarios, we set the total bandwidth of the spectrum W to be 6GHz, and it is equally divided into 201 subbands of bandwidth 30MHz. Among them, 30 subbands are occupied by PUs. Note that the spectrum is symmetric to the zero point, there are $J = 15$ independent active PUs, half the number of the occupied subbands. The PU occupation ratio is 15%.

1) Comparison of Sampling Rate:

In Table I, we compare the sampling rate between our scheme and existing ones. Here, we explain how our scheme

achieves same or better performance. In our setting, for Nyquist Sampling, the sampling rate should be twice of the one-sided bandwidth, which is 6GHz. For existing compressed spectrum sensing schemes [?]-[?], each node first obtains local sensing result. Thus, the dimension of the local measured vector should be about $4 \times$ the sparsity level to achieve exact recovery. This is the four-to-one practical rule well known to many CS researchers [?]. As a result, the sampling rate should be four times of the occupied bandwidth, which is 3.6GHz in our setting.

We should point out that although the required sampling rate decreased a lot with previous compressed spectrum sensing methods, it is still too high for the resource-constraint CRSN in the ultra-wideband regime. The novel idea of our scheme is to find a distributive compressed sampling structure, and further reduce the individual sampling rate. In our scheme, we effectively distribute the sensing load evenly to each CRSN nodes by splitting the compressed sampling process, and acquire the sensing result at the FC. Our individual sampling rate is f_s and the sum sampling rate for a cluster is $K \times f_s$. With the same CS recovery algorithm, the performance is mainly influenced by the sum sampling rate. In other words, the performance remains unchanged as long as $K \times f_s$ is a constant. The decrease in f_s will result in the increase of K , and vice versa. We can allocate individual sampling rates that the sum sampling rate of all CRSN nodes reaches four times of the occupied bandwidth. When $K \times f_s = 3.6\text{GHz}$, our scheme can achieve equivalent performance to existing compressed spectrum sensing schemes. For convenience of spectrum access, we choose the local sampling rate to be equal to the bandwidth of SU subband, i.e. $f_s = B = 30\text{MHz}$. As more CRSN nodes engage in the sensing task, more robust performance can be achieved.

For all the following Monte Carlo experiments, we repeat a hundred thousand times to compute the target value.

2) Performance of Compressed Sensing

In the first experiment, we examine the effect of measurement noise and K on the recovery accuracy in terms of MSE. As shown in Fig.4, we adjust the K from 25 to 50, and find that the MSE drops as K increases. It shows that, if too few measurements are collected, the performance of the compressed sensing algorithm will deteriorate.

3) Performance of Distributed Wideband Sensing

In the second experiment, we evaluate the performance of distributed wideband sensing in terms of P_d and P_f versus the number of engaged CRSN nodes K . We adjust K from 25 to 50, and find that P_d increases quickly, while P_f drops.

TABLE I
A COMPARISON OF SAMPLING RATE REQUIRED BY DIFFERENT SCHEMES

System Parameters	
Total Bandwidth	6 GHz
Subband Bandwidth	30 MHz
Occupied Bandwidth	900 MHz
Total Subbands	201
Occupied Subbands	30
Occupation Ratio	15%
Comparison of Sampling Rate at Individual Node	
Conventional Nyquist Sampling	6 GHz
Existing Compressed Spectrum Sensing	3.6 GHz
Distributed Compressed Wideband Sensing	30 MHz

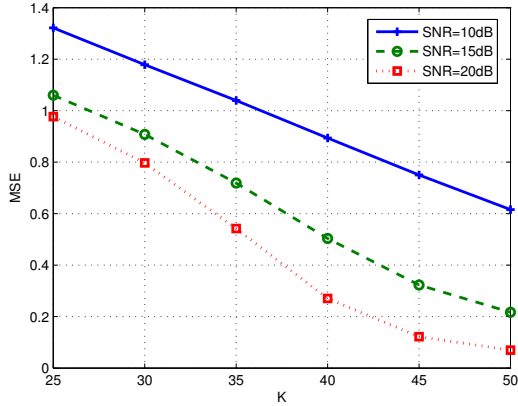


Fig. 4. MSE of compressed sensing vs K .

This implies that as more CRSN nodes engaged in the sensing task, the detecting performance of the cluster improves. The numerical results are shown in Fig.5.

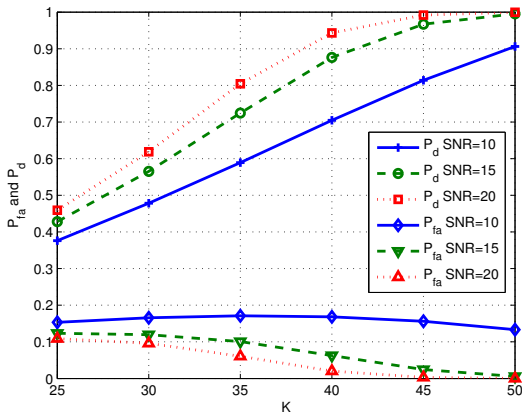


Fig. 5. Probability of detection and false alarm vs K .

We also computed the Receiver Operating Characteristic (ROC) curve under different K . As shown in Fig.6, the closer the curve is to the upper left corner, the better the performance is. The four curves represent the cases when CRSN nodes numbers are 25, 30, 40 and 60. The performance improves as

the engaging CRSN nodes number K grows. Interestingly, we observe that as K outnumbers J by four times, the detector's performance is nearly optimal.

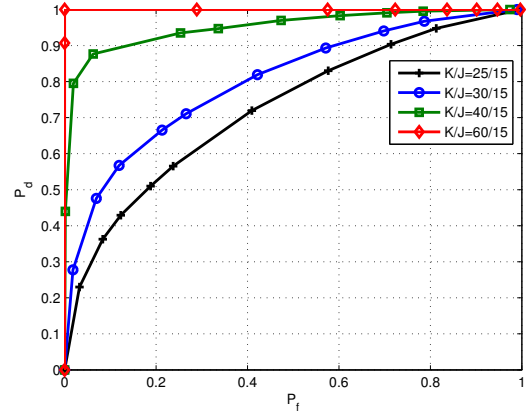


Fig. 6. ROC curve of the proposed detector.

V. CONCLUSION

In this paper, we proposed a distributed compressed wideband sensing scheme for the resource-constrained Cognitive Radio Sensor Networks (CRSN). First, we described the considered scenario by modeling the primary user signal and the clustered CRSN structure. Second, we provided a specific and practical structure for the sampler, which has very low sampling rate at individual node. Then, we introduced the sensing information fusion scheme, and described the compressed sensing recovery algorithm at the fusion center. Finally, we carried out several numerical simulations to validate our proposed scheme.

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